Lesson 4-4b

Fundamental Theorem Of Calculus II

Objective

Students will…

- Be able to know the Second Fundamental Theorem of Calculus.
- Be able to use the Second FTC to evaluate the derivative of a definite integral.

Fundamental Theorem of Calculus

Fundamental Theorem of Calculus- If a function f is continuous on the closed interval $[a, b]$, then $\int_a^b f(x)\,dx = F(b) - F(a)$

Recall that $F(x)$ is the **antiderivative** of $f(x)$.

This is clearly the most important theorem of all of Calculus (hence the word "fundamental"). Provided you can find an antiderivative of f, you now have a way to evaluate the area underneath the curve (instead of approximating) using **definite integral**.

Derivatives vs Antiderivatives

On the surface it is obvious to see how derivatives and antiderivatives are inverses of each other. Thus, differentiation and integration are inverses of each other, which can be quickly seen with indefinite integrals. For example…

$$\sin x = \cancel{\int}\cancel{\frac{d}{dx}}\sin x = \cancel{\frac{d}{dx}}\cancel{\int} \sin x$$

$$= \int \cos x$$

$$= \frac{d}{dx} -\cos x = \sin x.$$

But what about for **definite** integrals? Consider…

$$\frac{d}{dx}\left[\int_{\frac{\pi}{2}}^{x^3} \cos t\, dt\right] = \frac{d}{dx}\left[\sin t\,\Big|_{\pi/2}^{x^3}\right] = \frac{d}{dx}\left[\sin x^3 - \sin\frac{\pi}{2}\right]$$

$$= \frac{d}{dx}\left[\sin x^3 - 1\right] = \cos x^3 \cdot 3x^2$$

$$= \boxed{3x^2 \cos x^3}$$

Second Fundamental Theorem of Calculus

<u>Second Fundamental Theorem of Calculus</u>- If f is continuous on an open interval I containing a, then for every x in the interval,

$$\frac{d}{dx}\left[\int_a^x f(t)\,dt\right] = (f(x))(\frac{d}{dx}x)$$

In other words, the derivative of a definite integral is equal to the function of the upper bound times the derivative of the upper bound (chain rule!).

$$\frac{d}{dx}\int_1^2 f(x) = f(2)\cdot\frac{d}{dx}(2) = 0$$
$$\cdot\, 0$$

Example

Evaluate $\frac{d}{dx}\left[\int_0^x \sqrt{t^2+1}\,dt\right] = \sqrt{x^2+1} \cdot \frac{d}{dx}(x)$

$= \boxed{\sqrt{x^2+1}}$

Example

Evaluate $\frac{d}{dx}\left[\int_0^x \cos t \, dt\right]$ $= \cos x \cdot \frac{d}{dx}(x) = \boxed{\cos x}$.

Example

Evaluate $\frac{d}{dx}\left[\int_0^{x^7} \sin t \, dt\right]$ $= \sin x^7 \cdot \frac{d}{dx} x^7$

$= \boxed{7x^6 \sin x^7}$.

Homework 1/29

4.4 Exercises #5-21 (e.o.o), 39-41 (odd), 81-86