Warm Up 1/21

$0.4 \times 10 = 4$

Covert the following degree measures into radian.

1. $67°$

$67 \cdot \frac{\pi}{180} = \boxed{\frac{67\pi}{180}}$

2. $22°$

$\frac{22\pi}{180} = \boxed{\frac{11\pi}{90}}$

3. $299°$

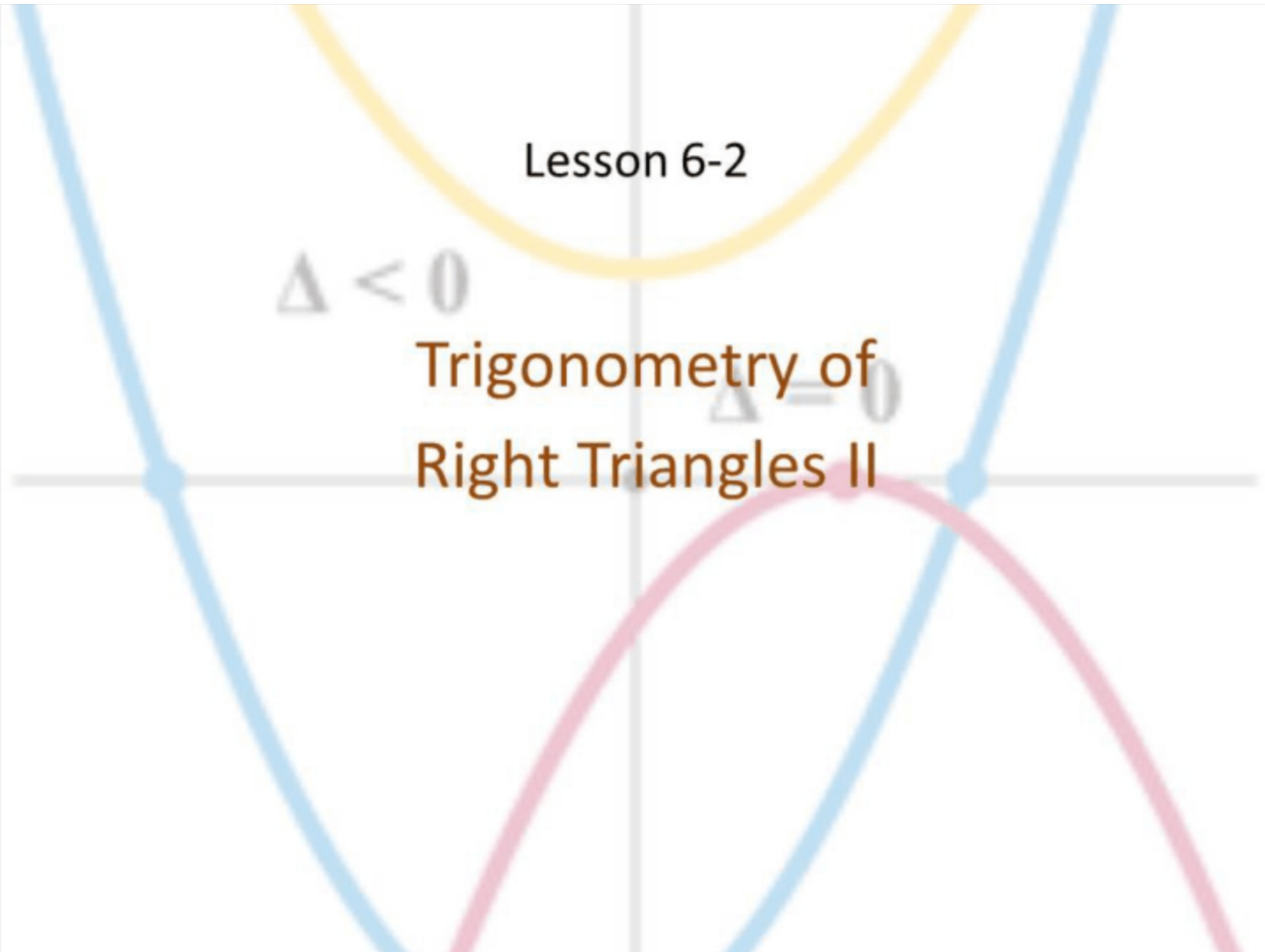
$\boxed{\frac{299\pi}{180}}$

4. $-112°$

$\frac{-112\pi}{180} = \boxed{\frac{-28\pi}{45}}$

5. $0.4°$

$\frac{0.4\pi \cdot 10}{180 \cdot 10} = \frac{4\pi}{1800} = \boxed{\frac{\pi}{450}}$


Lesson 6-2
Δ < 0
Trigonometry of
Δ = 0
Right Triangles II

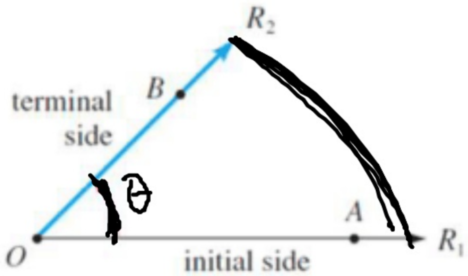
Objective

Students will…

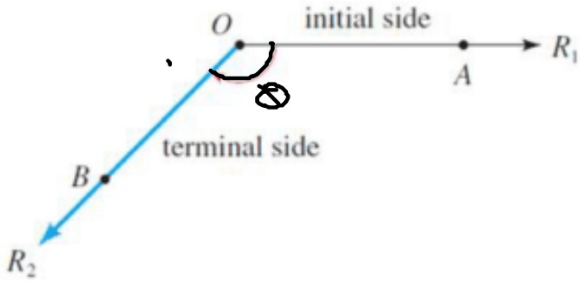
- Be able to calculate the length of a circular arc and circular sector area, given the radius of the circle and the angle measure.
- Be able to differentiate between angular and linear speed and compute them.

Angles

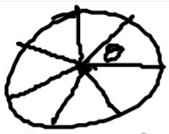
In trigonometry, however, angles are viewed as **rotations** of **one** line. In other words, angle measurement represents the **distance** travelled, or rotated. The beginning, or the stationary, position is known as the **initial side**, while the line at its finishing position is known as the **terminal side**. In this case, rotating **counter-clockwise** is positive, while rotating **clockwise** is negative.



Positive angle



Negative angle



Circular Arc Length

s.

In our last lesson, we learned that radian represented the **angle measurement** of the rotation, or the **distance travelled**. In this lesson, we now want to calculate the **linear** length of this distance. This is known as the **arc length**. Even though an arc is a part of a circle, we say linear length, since we can always picture cutting this arc out and laying it flat, or **straight**. We would then be able to measure the length with a ruler, for example. Following is the formula for measuring the arc length, ***s***, with radius, $\underline{r}$ and radian angle measurement $\underline{\theta}$:

$C = 2\pi r$

$\frac{\theta}{2\pi} 2\pi r = s$ $\quad \Rightarrow$

$$s = r\theta$$

We can then modify this equation and get a very important formula:

$$\theta = \frac{s}{r}$$

One thing to keep in mind is we always need to use **radians**.

Example

Find the length of an arc of a circle with radius 10m that subtends a central angle of 30°. $S = r\theta$

$30 \cdot \frac{\pi}{180} = \frac{\pi}{6}$ $S = 10\left(\frac{\pi}{6}\right) = \boxed{\frac{5\pi}{3}}$

$40 \cdot 2 =$

Find the length of an arc of a circle with radius 21m that subtends a central angle of 15°. $= \frac{\pi}{12}$

$\frac{7\pi}{4}$ vs $\cancel{\frac{7\pi}{8}}$

$s = r\theta$

$s = 21\left(\frac{\pi}{12}\right) = \boxed{\frac{7\pi}{4}}$

A central angle θ in a circle of radius 4m is subtended by an arc of length 6m. Find the measure of θ in radians.

$$s = r\theta$$

$$\frac{s}{r} = \theta$$

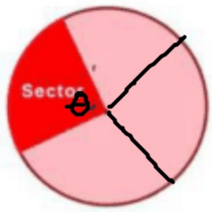
$$\theta = \frac{6}{4} = \boxed{\frac{3}{2}}$$

A central angle θ in a circle of radius 9m is subtended by an arc of length 12m. Find the measure of θ in radians.

$$\theta = \frac{s}{r} = \frac{12}{9} = \boxed{\frac{4}{3}}$$

Area of a Circular Sector

We can also find the area of a **circular sector** by any given central angle θ. The section in red is the circular sector.



Combining with the area formula of a circle: $A = \pi r^2$, we get the following formula for finding the area of a given circular sector.

$\frac{\theta}{2\pi} \cdot \cancel{\pi} r^2 = \cancel{\frac{\theta 2\pi}{2\pi\theta}}$ (SA)

$\frac{1}{2} r^2 \theta$

radians

$$\boxed{A = \frac{1}{2} r^2 \theta}$$

Ex. Find the area of a sector of a circle with central angle $60°$ if the radius of the circle is 3m.

$60 \cdot \frac{\pi}{180} = \frac{\pi}{3}$

$A = \frac{1}{2}(3^2)\left(\frac{\pi}{3}\right) = \frac{3\cancel{9}}{2}\left(\frac{\pi}{1\cancel{3}}\right) = \boxed{\frac{3\pi}{2}}$

Circular Motion

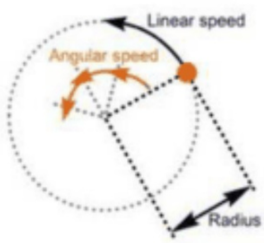
The last thing regarding the angular rotation is the concept of **circular motion**, which simply describes an object moving in circles. There are two ways to describe this type of motion: **linear** and **angular** speed.

Linear speed describes the **explicit** distance travelled over **time** (i.e. how fast the object is traveling along the circle). The unit is $\frac{\textbf{\textit{distance}}\ (\textbf{\textit{m,miles,etc.}})}{\textbf{\textit{time}}}$

Linear Speed ($\boldsymbol{v}$): $\boldsymbol{v} = \frac{\boldsymbol{s}}{\boldsymbol{t}}$, where s is the **arc length**.

Angular speed, on the other hand, describes the **angular change** over time (i.e. how fast the angle is changing). The unit is $\frac{\textbf{\textit{angle}}\ (\textbf{\textit{rad or deg}})}{\textbf{\textit{time}}}$

Angular Speed ($\boldsymbol{\omega}$): $\boldsymbol{\omega} = \frac{\boldsymbol{\theta}}{\boldsymbol{t}}$



$15 \cdot 2\pi$
$\theta = 30\pi$

Example

$=r$

A boy rotates a stone in a 3-ft-long sling at the rate of 15 revolutions every 10 seconds. Find the angular and linear speeds of the stone.

ang. : $\omega = \frac{\theta}{t}$

$\omega = \frac{30\pi}{10}$

$\boxed{\omega = 3\pi}$

lin : $v = \frac{s}{t}$

$v = \frac{90\pi}{10}$

$\boxed{V = 9\pi}$

$s = r\theta$

$s = 3(30\pi)$

$s = 90\pi$

A disk with a 12-inch diameter spins at the rate of 45 revolutions per minute. Find the angular and linear velocities of a point at the edge of the disk in radians per second and inches per second, respectively.

$\theta = 2\pi \cdot 45$

$\theta = 90\pi$

$\omega = \frac{90\pi}{60}$

$= \boxed{\frac{3\pi}{2}}$

$V = \frac{3\pi}{2} \cdot \frac{6}{1} = \boxed{9\pi}$

Linear and Angular Speed

It turns out that there is a way to take any angular speed and find its corresponding linear speed. With v being the linear speed, and ω being the angular speed, with radius r we have the following:

$$v = r\omega$$

Ex. A woman is riding a bicycle whose wheels are 26 inches in diameter. If the wheels rotate at 125 rpm (revolutions per minute), find the speed at which she is traveling.

A woman is riding a bicycle whose wheels are 30 inches in diameter. If the wheels rotate at 150 rpm (revolutions per minute), find the speed at which she is traveling.

Homework 1/21

TB pg. 475 #49-51, 53, 59, 79, 81