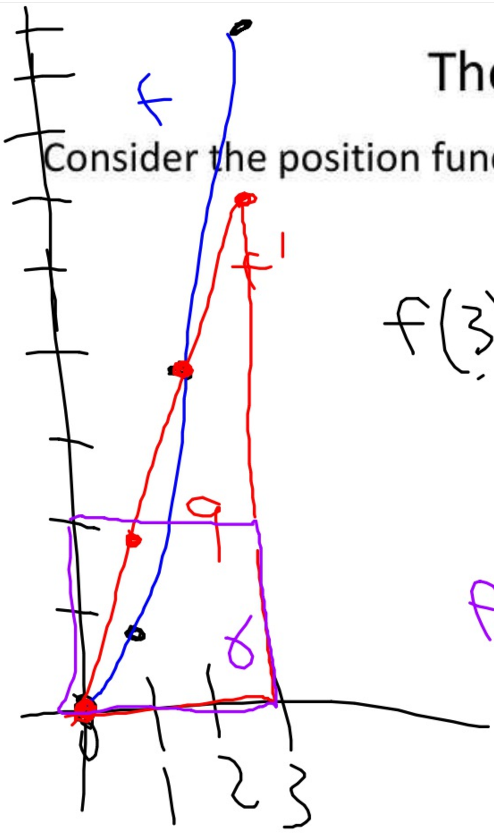
Lesson 4-4

Fundamental Theorem Of Calculus

Objective

Students will…

- Be able to know the Fundamental Theorem of Calculus.
- Be able to use the FTC to find the area underneath the curve.



The Velocity Problem

Consider the position function ~~$f(x) = x^2$~~ $f(t) = t^2$, t = seconds, $f(t)$ = ft.

$f'(t) = 2t$ t = seconds, $f'(t)$ = ft/sec.

$f(3) - f(0)$ = Area of $f'(t)$ on $[0, 3]$.

$f'(3) - f'(0) = 6 - 0 = 6$

$f''(t) = 2$

$\sum_{i=1}^{b}$

Fundamental Theorem of Calculus

Fundamental Theorem of Calculus- If a function f is continuous on the closed interval $[a, b]$, then $\int_a^b f(x)\,dx = F(b) - F(a)$

Recall that $F(x)$ is the **antiderivative** of $f(x)$.

This is clearly the most important theorem of all of Calculus (hence the word “fundamental”). Provided you can find an antiderivative of f, you now have a way to evaluate the area underneath the curve (instead of approximating) using **definite integral**.

Example

Use five rectangles to find the area of the region bounded by $f(x) = x^2$, the x-axis and $x = 0$ and $x = 10$.

$$\text{Area} = \int_0^{10} x^2\,dx = \frac{1}{3}x^3 \Big|_0^{10} = \frac{1}{3}(10)^3 - \cancel{\frac{1}{3}(0)^3}$$

$$= \frac{1000}{3} = 333.\overline{333}\ldots = \boxed{333\frac{1}{3}}$$

Left: 240

Right: 440

midpt: 330

trap: 340

Example

Use five rectangles to find the area of the region bounded by $f(x) = -x^2 + 5$, the x-axis and $x = 0$ and $x = 2$.

Area: $\int_0^2 -x^2+5 = -\frac{1}{3}x^3+5x \Big|_0^2$

$= -\frac{1}{3}(2)^3+5(2) - 0$

$= \frac{-8}{3} + \frac{30}{3} = \boxed{\frac{22}{3}}$

Example

~~Use six rectangles to~~ find the area of the region bounded by $f(x) = \sin x$, the x-axis and $x = 0$ and $x = \pi$

$$\int_0^{\pi} \sin x \, dx = -\cos x \Big|_0^{\pi} = 1 - -1 = \boxed{2}$$

Homework 1/17

Previous WKSHT use the definite integral to evaluate the actual area.