Lesson 4-6

Midpoint and Trapezoidal Rule

Objective

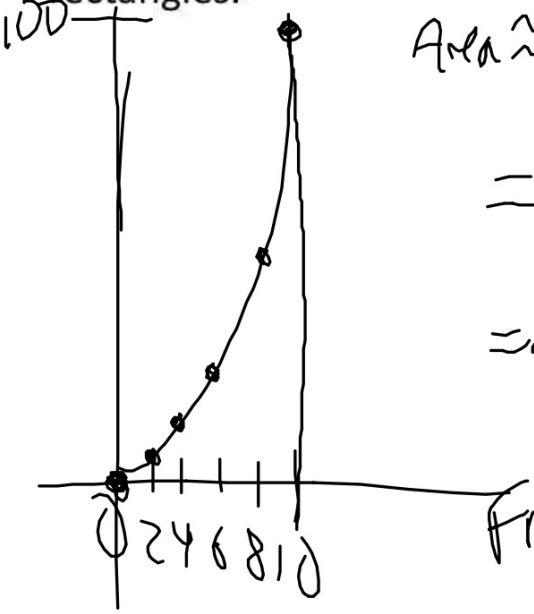
Students will…

- Be able to approximate the Riemann Sum using the Midpoint Rule
- Be able to approximate the Riemann Sum using the Trapezoidal Rule.

Midpoint Rule

$\frac{b-a}{n} = \frac{10-0}{5} = 2$

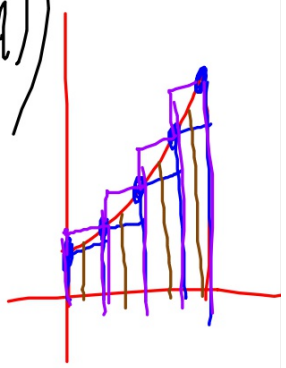
Consider the following problem: For $f(x) = x^2$, approximate the area of the region bounded by the x-axis between $x = 0$ and $x = 10$, using 5 rectangles. $n = 5$



$$\text{Area} \approx 2\left(f(1) + f(3) + f(5) + f(7) + f(9)\right) \quad (a, b)$$

$$= 2(1 + 9 + 25 + 49 + 81)$$

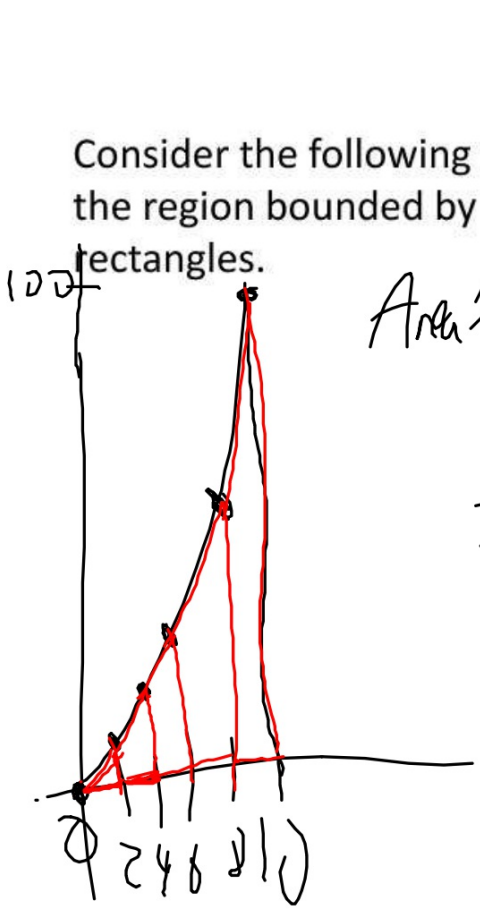
$$= 2(165) = \boxed{330}$$ ✓



From Previous: Left: 240
Right: 440

Trapezoidal Rule

$\frac{1}{2}h(b_1+b_2)$



Consider the following problem: For $f(x) = x^2$, approximate the area of the region bounded by the x-axis between $x = 0$ and $x = 10$, using 5 rectangles.

$$\text{Area} \approx \frac{1}{2}(2)(0+4+4+16+16+36+36+64+64+100)$$

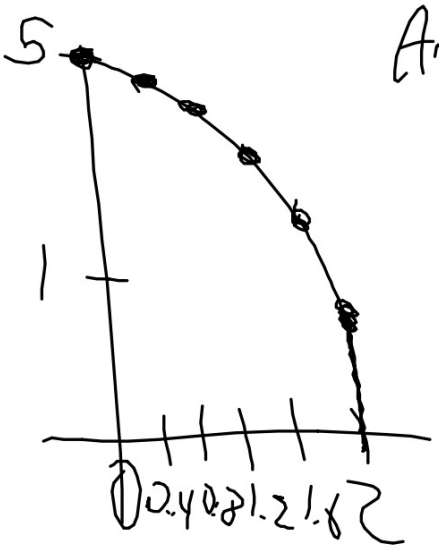
$$= 1(340) = \boxed{340}$$

Left: 240
Mid: 330
Trap: 340
Right: 440

Example

$\frac{b-a}{n} = \frac{2-0}{5} = \frac{2}{5} = 0.4$

Use five rectangles to find the trapezoidal Riemann Sums of $f(x) = -x^2 + 5$ bounded by the x-axis and $x = 0$ and $x = 2$



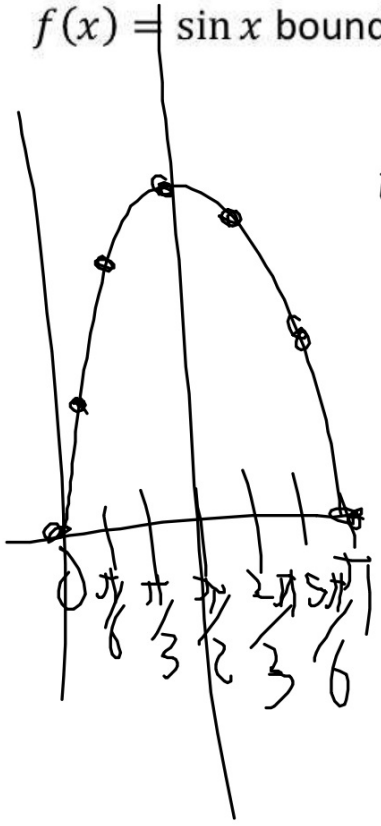
$$\text{Area} \approx \frac{1}{2}(0.4)\left(f(0) + 2f(0.4) + 2f(0.8) + 2f(1.2) + 2f(1.6) + f(2)\right)$$

$$= \frac{1}{2}(0.4)\left(5 + 2(4.84) + 2(4.36) + 2(3.56) + 2(2.44) + 1\right)$$

$$\approx 7.28$$

Example

Use six rectangles to find the trapezoidal Riemann Sums of $f(x) = \sin x$ bounded by the x-axis and $x = 0$ and $x = \pi$



$$\text{Area} \approx \frac{1}{2}\left(\frac{\pi}{6}\right)\left(0 + 2\left(\frac{1}{2}\right) + 2\left(\frac{\sqrt{3}}{2}\right) + 1\right)$$

$$\frac{\pi}{6}\left(0 + 1 + \sqrt{3} + 1\right) = \frac{\pi}{6}\left(2 + \sqrt{3}\right)$$

$$= \frac{\pi}{3} + \frac{\pi\sqrt{3}}{6}$$

Homework 1/16

Previous WKSHT (use midpoint rule for the front, trapezoidal for the back)