

## Warm Up 12/10

Find the period and the amplitude of the following.

1.  $2\sin 4\pi$

2.  $5 \cos \frac{1}{2}\pi$

3.  $7 \sin 0.6\pi$

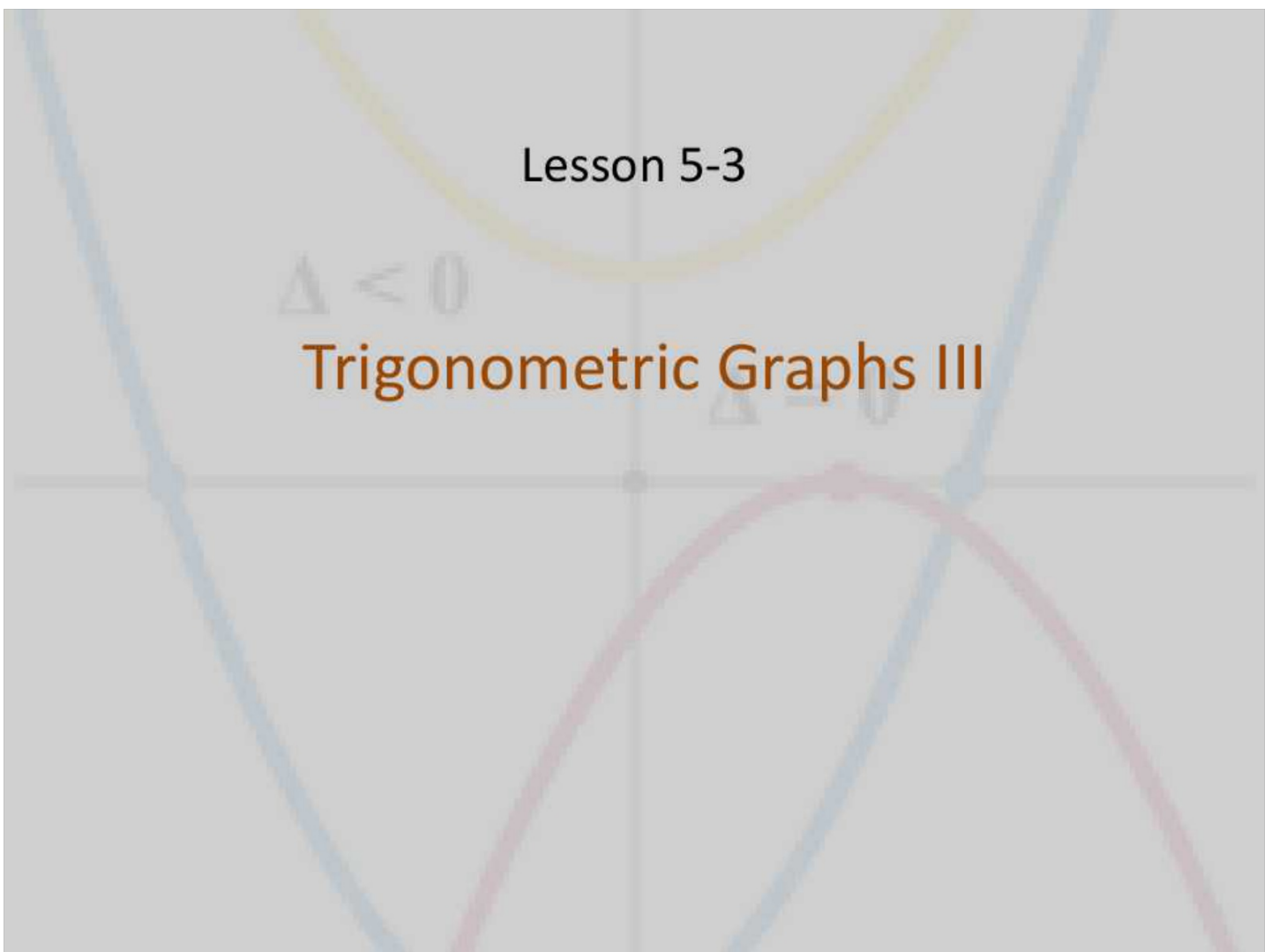
4. What does it mean for a function to have a period of  $3\pi$ ?

Lesson 5-3

$\Delta < 0$

Trigonometric Graphs III

$\Delta = 0$



## Objective

Students will...

- Be able to identify and graph the shift of sine and cosine functions.

## Standard Equation of Sine and Cosine Curves

Like any other functions, there exists a standard equation of both sine and cosine curves.

**Sine Curves**: Any equation of a sine curve is written in the form:

$$y = a \sin kx, \text{ where } a \text{ and } k \text{ are real numbers with } k > 0$$

$$y = a \sin k(x-h) + m$$

**Cosine Curves**: Any equation of a cosine curve is written in the form:

$$y = a \cos kx, \text{ where } a \text{ and } k \text{ are real numbers with } k > 0$$

$$y = a \cos k(x-h) + m$$

## Period and Amplitude of Sine and Cosine Curves

In our previous lesson we simply used the graph to figure out the period and amplitude of a given sine or cosine curve. However, we may not (more of than not) have a graph to refer to. In fact, how would we find the period if we were asked to graph a given sine or cosine curve? Of course, we can use the x-y table to graph the curve first, but this isn't always practical.

Fortunately, finding the period and the amplitude of a sine or cosine curve can be found algebraically from their equation.

For sine and cosine curves:  $y = a \sin kx$       and       $y = a \cos kx$ ,

$$\text{Period} = \frac{2\pi}{k}$$

$$\text{Amplitude} = |a|$$

## Horizontal and Vertical Shift

Recall from chapter 2 about the shift of parabolas. The standard equation of a parabola is  $y = x^2$ . Now, consider...

Ex.

$$y = x^2$$

$$y = (x - 4)^2 - 9$$

**Vertex:**

$$(0, 0)$$

$$(4, -9)$$

**Shift:**

right 4, down 9

## Horizontal and Vertical Shift

Believe it or not, trig functions (along with many other functions) take the similar format when it comes to their shifts.

Ex.

$$y = |\cos x$$

**Period:**

$$2\pi$$

**Amplitude:**

$$1$$

**Shift:**

**Start/End Point:**

X-axis:  $0 \rightarrow 2\pi$  mid = 0

Y-axis:  $-1 \rightarrow 1$

$$y = |\cos^{k=1}(x - \frac{\pi}{2}) + 1$$

$$2\pi$$

$$1$$

right  $\pi/2$ , up 1

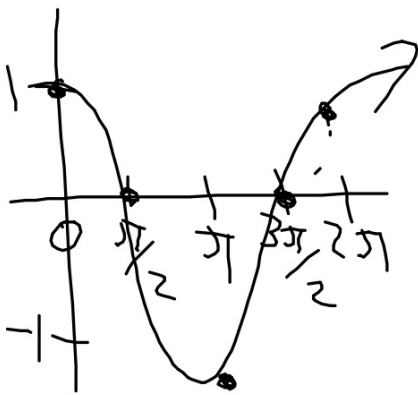
X-axis:  $\pi/2 \rightarrow \frac{5\pi}{2}$

Y-axis:  $0 \rightarrow 2$  mid = 1

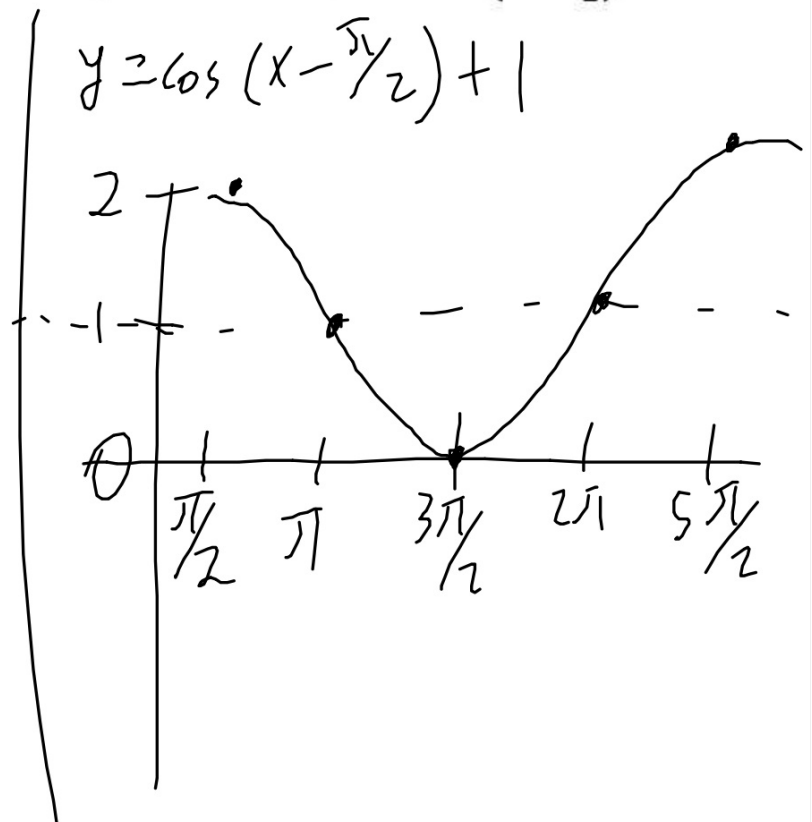
## Examples

Let's graph the two and compare.  $y = \cos x$ ,  $y = \cos\left(x - \frac{\pi}{2}\right) + 1$

$$y = \cos x$$



$$y = \cos\left(x - \frac{\pi}{2}\right) + 1$$





## Example

Believe it or not, trig functions (along with many other functions) take the similar format when it comes to their shifts.

Ex.

**Period:**

**Amplitude:**

**Shift:**

**Start/End Point:**

$$\begin{aligned} \text{X-axis: } & 0 \rightarrow 2\pi \\ \text{Y-axis: } & -1 \rightarrow 1 \text{ mid} = 0 \end{aligned}$$

$$y = \sin x$$

$$2\pi$$

$$1$$

$$3 \sin\left(2x - \frac{2\pi}{4}\right)$$

$$y = 3 \sin^k\left(x - \frac{\pi}{4}\right)$$

$$\frac{2\pi}{k} = \frac{2\pi}{2} = \pi$$

$$3$$

right  $\pi/4$

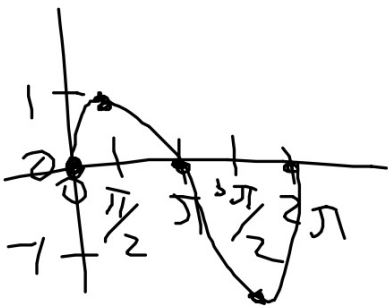
X-axis: From:  $0 \rightarrow \pi$   
w/shift:  $\pi/4 \rightarrow 5\pi/4$

Y-axis:  $-3 \rightarrow 3$  mid = 0

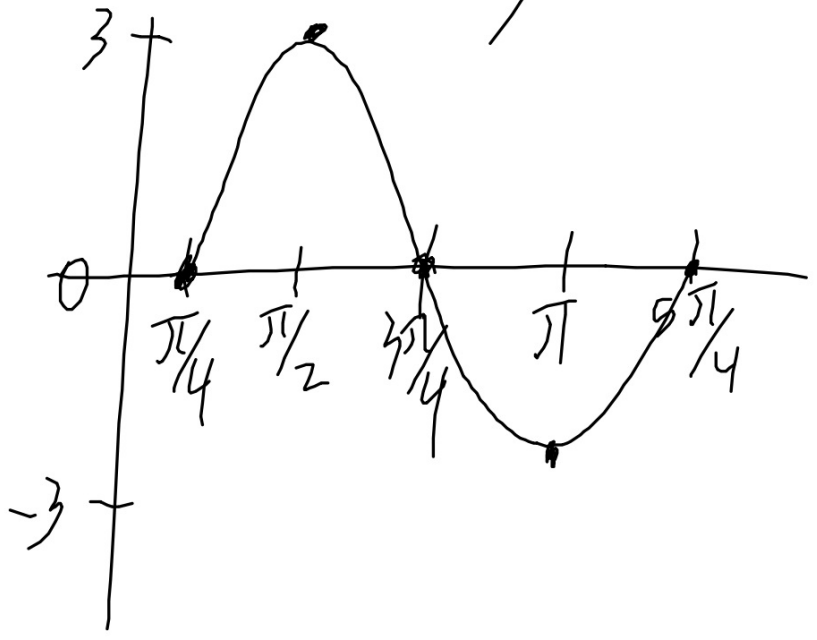
## Examples

Let's graph the two and compare.  $y = \sin x$ ,  $y = 3 \sin 2\left(x - \frac{\pi}{4}\right)$

$$y = \sin x$$



$$y = 3 \sin 2\left(x - \frac{\pi}{4}\right)$$



## Guidelines to Graphing

1. Identify whether it is a sine or a cosine function.
2. Find the period and the amplitude.  
 $\frac{2\pi}{k}$   $|a|$
3. Find the phase shift of the functions.  
(left/right, up/down)
4. Identify the starting points and the endpoints of the shifted graph.
5. Graph

↑  
middle

## Examples

Graph the following (pg. 429)

1.  $f(x) = 1 + \cos x$

## Examples

Graph the following (pg. 429)

$$33. y = 5 \cos\left(3x - \frac{\pi}{4}\right) \frac{1}{3}$$

$$\text{Period: } \frac{2\pi}{3}$$

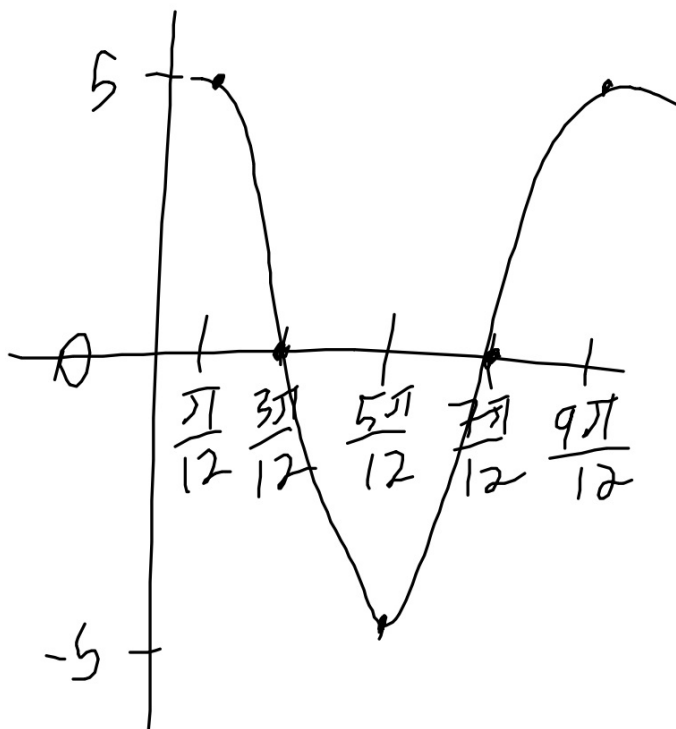
$$\text{Amp: } 5 \quad \text{Shift: right } \frac{\pi}{12}$$

Start/End:

$$\underline{\text{X-axis}}: \frac{\pi}{12} \rightarrow \frac{9\pi}{12}$$

$$\underline{\text{Y-axis}}: -5 \rightarrow 5$$

mid = 0



## Homework 12/10

TB pg. 429 #1, 11, 19, 27, 33, 36

**(Be sure to graph!)**