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9. $(\sin\theta+\cos\theta)^2+(\sin\theta-\cos\theta)^2=2$

LHS $=\sin^2\theta+2\sin\theta\cos\theta+\cos^2\theta+\sin^2\theta-2\sin\theta\cos\theta+\cos^2\theta$

$=2\sin^2\theta+2\cos^2\theta=2(\sin^2\theta+\cos^2\theta)$

$=2(1)=2=$ RHS ✓

10. $(\sin\theta+\cos\theta)(\tan\theta+\cot\theta)=\sec\theta+\csc\theta$

LHS $=\sin\theta\tan\theta+\sin\theta\cot\theta+\cos\theta\tan\theta+\cos\theta\cot\theta$

$=\frac{\sin\theta}{1}\cdot\frac{\sin\theta}{\cos\theta}+\frac{\sin\theta}{1}\cdot\frac{\cos\theta}{\sin\theta}+\frac{\cos\theta}{1}\cdot\frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{1}\cdot\frac{\cos\theta}{\sin\theta}$

$=\left(\frac{\sin^2\theta}{\cos\theta}+\frac{\cos\theta}{1}\right)+\left(\frac{\sin\theta}{1}+\frac{\cos^2\theta}{\sin\theta}\right)$ [Group them]

$=\frac{\sin^2\theta+\cos^2\theta}{\cos\theta}+\frac{\sin^2\theta+\cos^2\theta}{\sin\theta}=\frac{1}{\cos\theta}+\frac{1}{\sin\theta}=\sec\theta+\csc\theta$ ✓ =RHS

11. $\frac{\tan\theta-1}{\tan\theta+1}=\frac{1-\cot\theta}{1+\cot\theta}$ (Conjugate)

RHS $=\frac{1-\frac{1}{\tan\theta}}{1+\frac{1}{\tan\theta}}=\frac{\frac{\tan\theta-1}{\tan\theta}}{\frac{\tan\theta+1}{\tan\theta}}=\frac{\tan\theta-1}{\tan\theta}\cdot\frac{\tan\theta}{\tan\theta+1}$

$=\frac{\tan\theta-1}{\tan\theta+1}=$ LHS ✓

12. $\frac{1-\tan^2x}{1+\tan^2x}=1-2\sin^2x$

LHS $=\frac{1-\tan^2x}{\sec^2x}=\frac{1-\frac{\sin^2x}{\cos^2x}}{\frac{1}{\cos^2x}}=\frac{\frac{\cos^2x-\sin^2x}{\cos^2x}}{\frac{1}{\cos^2x}}$

$=\frac{\cos^2x-\sin^2x}{\cos^2x}\cdot\frac{\cos^2x}{1}=\cos^2x-\sin^2x=(1-\sin^2x)-\sin^2x$ ✓

$=1-2\sin^2x=$ RHS

13. $\frac{\cos x+1}{\sin^3x}=\frac{\csc x}{1-\cos x}$

RHS $=\frac{\csc x}{1-\cos x}\cdot\frac{1+\cos x}{1+\cos x}=\frac{\csc x(1+\cos x)}{1-\cos^2x}$

$=\frac{\csc x(1+\cos x)}{\sin^2x}=\frac{1}{\sin x}\cdot\frac{1+\cos x}{\sin^2x}=\frac{1+\cos x}{\sin^3x}$

✓ $=$ LHS

14. $\csc^4x-\cot^4x=\csc^2x+\cot^2x$

diff. of 2 squares

LHS $=(\csc^2x+\cot^2x)(\csc^2x-\cot^2x)=(\csc^2x+\cot^2x)(1)$

$=\csc^2x+\cot^2x=$ RHS ✓

15. $\frac{\tan\theta}{\sec\theta}+\frac{\cot\theta}{\csc\theta}=\sin\theta+\cos\theta$

LHS $=\frac{\frac{\sin\theta}{\cos\theta}}{\frac{1}{\cos\theta}}+\frac{\frac{\cos\theta}{\sin\theta}}{\frac{1}{\sin\theta}}=\frac{\sin\theta}{\cos\theta}\cdot\frac{\cos\theta}{1}+\frac{\cos\theta}{\sin\theta}\cdot\frac{\sin\theta}{1}$

$=\sin\theta+\cos\theta=$ RHS ✓

16. $\frac{\sin y+\tan y}{1+\sec y}=\sin y$

$\frac{\sin y+\frac{\sin y}{\cos y}}{1+\frac{1}{\cos y}}=\frac{\frac{\sin y\cos y+\sin y}{\cos y}}{\frac{\cos y+1}{\cos y}}=$

$=\frac{\sin y\cos y+\sin y}{\cos y}\cdot\frac{\cos y}{\cos y+1}=\frac{\sin y(\cos y+1)}{\cos y+1}$

✓ $=\sin y=$ RHS

Trig Identities — Sheet 3.4 — name:

Prove each identity:

1. $\sec x - \tan x \sin x = \dfrac{1}{\sec x}$

$\frac{1}{\cos x} - \frac{\sin x}{\cos x}\frac{\sin x}{1}$

$\frac{1}{\cos x} - \frac{\sin^2 x}{\cos x}$

$\frac{1-\sin^2 x}{\cos x} = \frac{\cos^2 x}{\cos x} = \cos x$

RHS: $\cos x$ ✓

2. $\dfrac{1+\cos x}{\sin x} = \csc x + \cot x$

$RHS = \frac{1}{\sin x} + \frac{\cos x}{\sin x}$

$LHS = \frac{1+\cos x}{\sin x}$ ✓

3. $\dfrac{\sec\theta \sin\theta}{\tan\theta + \cot\theta} = \sin^2\theta$

$LHS = \frac{1}{\cos\theta}\cdot\sin\theta = \frac{\frac{\sin\theta}{\cos\theta}}{\tan\theta + \frac{1}{\tan\theta}} = \frac{\tan\theta}{\frac{\tan^2\theta+1}{\tan\theta}} = \frac{\tan\theta}{\sec^2\theta}$

$= \frac{\tan\theta}{1}\cdot\frac{\tan\theta}{\sec^2\theta} = \frac{\tan^2\theta}{\sec^2\theta} = \frac{\frac{\sin^2\theta}{\cos^2\theta}}{\frac{1}{\cos^2\theta}} = \sin^2\theta = RHS$ ✓

4. $\dfrac{\sec\theta}{\cos\theta} - \dfrac{\tan\theta}{\cot\theta} = 1$

$LHS = \frac{\frac{1}{\cos\theta}}{\cos\theta} - \frac{\frac{\sin\theta}{\cos\theta}}{\frac{\cos\theta}{\sin\theta}} = \frac{1}{\cos^2\theta} - \frac{\sin^2\theta}{\cos^2\theta}$

$= \frac{1-\sin^2\theta}{\cos^2\theta} = \frac{\cos^2\theta}{\cos^2\theta} = 1 = RHS$ ✓

5. $\cos^2 y - \sin^2 y = 1 - 2\sin^2 y$

$LHS = (1-\sin^2 y) - \sin^2 y = 1 - 2\sin^2 y = RHS$ ✓

6. $\csc^2\theta \tan^2\theta - 1 = \tan^2\theta$

$LHS = \frac{1}{\sin^2\theta}\cdot\frac{\sin^2\theta}{\cos^2\theta} - 1 = \frac{1}{\cos^2\theta} - 1$

$= \sec^2\theta - 1 = \tan^2\theta = RHS$ ✓

Remember: $\tan^2\theta + 1 = \sec^2\theta$

7. $\dfrac{\sec^2\theta}{\sec^2\theta - 1} = \csc^2\theta$

$LHS = \frac{\sec^2\theta}{\sec^2\theta - 1} = \frac{\sec^2\theta}{\tan^2\theta} = \frac{\frac{1}{\cos^2\theta}}{\frac{\sin^2\theta}{\cos^2\theta}} = \frac{1}{\cos^2\theta}\cdot\frac{\cos^2\theta}{\sin^2\theta}$

$= \frac{1}{\sin^2\theta} = \csc^2\theta = RHS$ ✓

8. $\tan^2 x \sin^2 x = \tan^2 x - \sin^2 x$

$RHS = \frac{\sin^2 x}{\cos^2 x} - \frac{\sin^2 x}{1} = \frac{\sin^2 x}{\cos^2 x} - \frac{\sin^2 x \cos^2 x}{\cos^2 x}$

$= \frac{\sin^2 x - \sin^2 x\cos^2 x}{\cos^2 x} = \frac{\sin^2 x(1-\cos^2 x)}{\cos^2 x} = \frac{\sin^2 x(\sin^2 x)}{\cos^2 x}$

$= \left(\frac{\sin^2 x}{1}\right)\left(\frac{\sin^2 x}{\cos^2 x}\right) = \sin^2 x \tan^2 x = \tan^2 x \sin^2 x$

$= LHS$ ✓