Warm Up 2/17

Verify the identity.
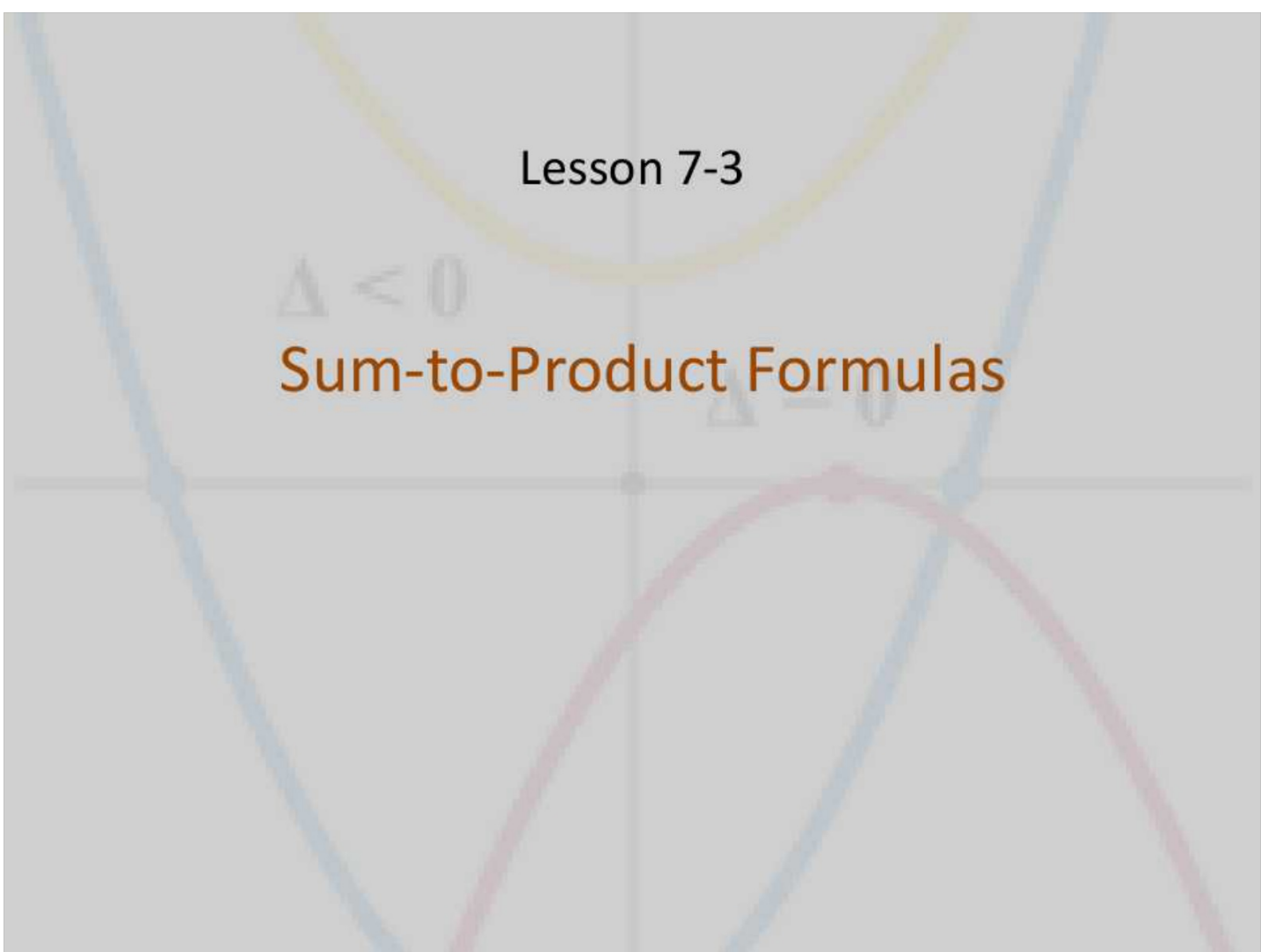
$$\cos(x + y) \cos(x - y) = \cos^2 x - \sin^2 y$$

Lesson 7-3

Sum-to-Product Formulas

Objective

Students will...

- Be able to know the Sum-to-Product Formulas.
- Be able to use the Sum-to-Product formulas to prove identities.

Sum-to-Product Formulas

We now move further into different formulas for trig functions. The following is the **Sum-to-Product** formulas, which do exactly as it says-turn sums (addition or subtraction) into products (multiplication).

Sum-to-Product Formulas:

$$\sin x + \sin y = 2 \sin \frac{x+y}{2} \cos \frac{x-y}{2}$$

$$\sin x - \sin y = 2 \cos \frac{x+y}{2} \sin \frac{x-y}{2}$$

$$\cos x + \cos y = 2 \cos \frac{x+y}{2} \cos \frac{x-y}{2}$$

$$\cos x - \cos y = 2 \sin \frac{x+y}{2} \sin \frac{x-y}{2}$$

Using Sum-to-Product Formulas

Write $\sin 7x + \sin 3x$ as a product.

$$= 2\sin\left(\frac{7x+3x}{2}\right)\cos\left(\frac{7x-3x}{2}\right)$$

$$= 2\sin(5x)\cos(2x)$$

Write $\sin 11x + \sin 5x$ as a product.

$$= 2\sin(8x)\cos(3x)$$

Homework Problem

Write the sum as a product.

49. $\cos 4x - \cos 6x$

$$= 2 \sin\left(\frac{4x+6x}{2}\right) \sin\left(\frac{4x-6x}{2}\right)$$

$$= 2 \sin(5x) \sin(-x)$$

$$= -2 \sin(5x) \sin(x).$$

Using Sum-to-Product Formulas

Verify the identity: $\dfrac{\sin 3x - \sin x}{\cos 3x + \cos x} = \tan x$

$$LHS: \frac{2\cos\left(\frac{3x+x}{2}\right)\sin\left(\frac{3x-x}{2}\right)}{2\cos\left(\frac{3x+x}{2}\right)\cos\left(\frac{3x-x}{2}\right)} = \frac{\sin(x)}{\cos(x)} = \tan x = RHS \checkmark$$

Using Sum-to-Product Formulas

Verify the identity: $\dfrac{\sin 4x + \sin 2x}{\sin 2x} = \dfrac{\sin 3x}{\sin x}$

$$\text{LHS} = \frac{2\sin\left(\frac{6x}{2}\right)\cos\left(\frac{2x}{2}\right)}{\sin x \cos x + \cos x \sin x}$$

$\overset{\sin(x+x)}{}$

$$= \frac{2\sin 3x \cos x}{2 \sin x \cos x}$$

$$= \frac{\sin 3x}{\sin x} = \text{RHS} \checkmark$$

Homework Problems

Verify the identity: 71. $\dfrac{\sin x + \sin 5x}{\cos x + \cos 5x} = \tan 3x$

Homework Problems

79. Show that $\sin 45° + \sin 15° = \sin 75°$

Homework 2/17

TB pg. 548-549 #47-53 (odd), 72, 74, 75, 77