

$$27) \tan(x - \pi) = \tan x$$

$$\underline{\text{LHS}}: \frac{\tan x - \tan \pi}{1 + \tan x \tan \pi} = \frac{\tan x - 0}{1 + \tan x(0)} = \tan x = \text{RHS} \quad \checkmark$$

$$39) \sin(x+(y+z)) = \sin x \cos y \cos z + \cos x \sin y \cos z \\ + \cos x \cos y \sin z - \sin x \sin y \sin z$$

LHS: $\sin x \cos(y+z) + \cos x \sin(y+z)$

$$= \sin x (\cos y \cos z - \sin y \sin z) + \cos x (\sin y \cos z + \cos y \sin z)$$

$$= \sin x \cos y \cos z - \sin x \sin y \sin z + \cos x \sin y \cos z + \cos x \cos y \sin z$$

= RHS



$$37) \frac{\sin(x+y) - \sin(x-y)}{\cos(x+y) + \cos(x-y)} = \tan y$$

$$\text{LHS: } \frac{\sin x \cos y + \cos x \sin y - \sin x \cos y + \cos x \sin y}{\cos x \cos y - \sin x \sin y + \cos x \cos y + \sin x \sin y}$$

$$\frac{2 \cos x \sin y}{2 \cos x \cos y}$$

$$= \frac{2 \cos x \sin y}{2 \cos x \cos y} = \frac{\sin y}{\cos y} = \tan y = \text{RHS} \quad \checkmark$$

$$35) \tan x - \tan y = \frac{\sin(x-y)}{\cos x \cos y}$$

$$\underline{\text{RHS}}: \frac{\sin x \cos y - \cos x \sin y}{\cos x \cos y}$$

$$= \frac{\cancel{\sin x} \cos y}{\cos x \cancel{\cos y}} - \frac{\cos x \cancel{\sin y}}{\cancel{\cos x} \cos y}$$

$$= \tan x - \tan y$$

$$= \text{LHS} \quad \checkmark$$

$$\Rightarrow \sin\left(x - \frac{\pi}{2}\right) = -\cos x$$

$$\underline{\text{LHS}}: \sin x \cos \frac{\pi}{2} - \cos x \sin \frac{\pi}{2}$$

$$= -\cos x = \text{RHS} \checkmark$$

$$31) \sin(x+y) - \sin(x-y) = 2\cos x \sin y$$

$$\underline{\text{LHS}}: \sin x \cos y + \cos x \sin y - \sin x \cos y + \cos x \sin y$$

$$= 2\cos x \sin y = \text{RHS} \quad \checkmark$$