Warm Up 2/26
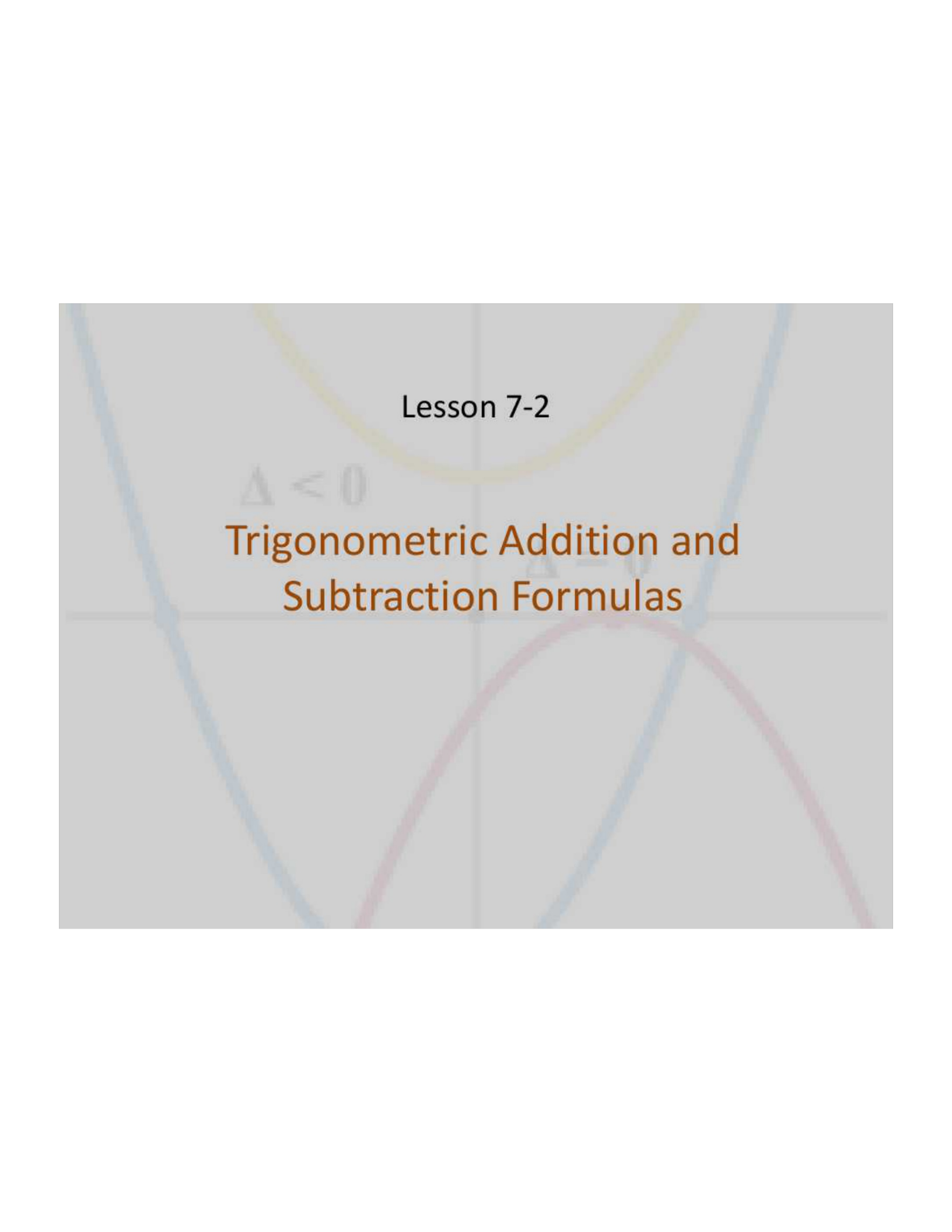
Verify the identity.

$$\frac{\sec x}{\sec x - \tan x} = \sec x (\sec x + \tan x)$$

LHS: $\dfrac{\frac{1}{\cos x}}{\frac{1}{\cos x} - \frac{\sin x}{\cos x}} = \dfrac{\frac{1}{\cos x}}{\frac{1-\sin x}{\cos x}} \cdot \dfrac{\cos x}{1-\sin x} = \dfrac{1}{1-\sin x}$

RHS: $\sec^2 x + \tan x \sec x = \dfrac{1}{\cos^2 x} + \dfrac{\sin x}{\cos^2 x} = \dfrac{1+\sin x}{\cos^2 x}$

$= \dfrac{1+\sin x}{1-\sin^2 x} = \dfrac{1+\sin x}{(1+\sin x)(1-\sin x)} = \dfrac{1}{1-\sin x}$ ✓

Lesson 7-2

Trigonometric Addition and Subtraction Formulas

Objective

Students will...

- Be able to know the addition and subtraction formulas for sine, cosine, and tangent.
- Be able to use addition and subtraction formulas to evaluate trig functions and to prove or verify identities.

Trigonometric Identities

Try to fill these in from memory as much as possible!

$$\csc x = \frac{1}{\sin x} \qquad\qquad \sec x = \frac{1}{\cos x} \qquad\qquad \cot x = \frac{1}{\tan x}$$

$$\tan x = \frac{\sin x}{\cos x} \qquad\qquad \cot x = \frac{\cos x}{\sin x}$$

Pythagorean Identity: $\sin^2 x + \cos^2 x = 1$

From this, we also get:

$\sin^2 x = 1 - \cos^2 x$ and $\cos^2 x = 1 - \sin^2 x$

$\tan^2 x + 1 = \sec^2 x$ and $1 + \cot^2 x = \csc^2 x$

$\tan^2 x = \sec^2 x - 1$ and $\cot^2 x = \csc^2 x - 1$

$\sec^2 x - \tan^2 x = 1$ and $\csc^2 x - \cot^2 x = 1$

Addition and Subtraction Formulas

Formulas for Sine:

$$\sin(s + t) = \sin s \cos t + \cos s \sin t$$
$$\sin(s - t) = \sin s \cos t - \cos s \sin t$$

Formulas for Cosine:

$$\cos(s + t) = \cos s \cos t - \sin s \sin t$$
$$\cos(s - t) = \cos s \cos t + \sin s \sin t$$

Formulas for Tangent:

$$\tan(s + t) = \frac{\tan s + \tan t}{1 - \tan s \tan t}$$

$$\tan(s - t) = \frac{\tan s - \tan t}{1 + \tan s \tan t}$$

Using Addition and Subtraction Formulas

Find the **exact** value of each expression.

a) $\cos 75°$

$= \cos(45+30)$

$= \cos 45 \cos 30$

$-$

$\sin 45 \sin 30$

$= \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)$

$= \dfrac{\sqrt{6} - \sqrt{2}}{4}$

b) $\cos\frac{\pi}{12} = \cos\left(\frac{\pi}{3} - \frac{\pi}{4}\right)$

$= \cos\frac{\pi}{3}\cos \frac{\pi}{4}$

$+$

$\sin \frac{\pi}{3} \sin \frac{\pi}{4}$

$\left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$

$\dfrac{\sqrt{2} + \sqrt{6}}{4}$

c) $\cos 15° = \cos(45 - 30)$

or

$\cos(60 - 45)$

Example

Find the exact value of the expression:

$$\sin 20° \cos 40° + \cos 20° \sin 40°$$

Example

Find the exact value of the expression:

$\sin 10° \cos 50° + \cos 10° \sin 50°$

Homework Problems

Find the **exact** value of each expression.

3. $\cos 105°$

9. $\tan\left(-\dfrac{\pi}{12}\right)$

Homework Problems

Find the **exact** value of each expression.

15. $\cos \dfrac{3\pi}{7} \cos \dfrac{2\pi}{21} + \sin \dfrac{3\pi}{7} \sin \dfrac{2\pi}{21}$

Homework Problems

Find the **exact** value of each expression.

16. $\dfrac{\tan\frac{\pi}{18}+\tan\frac{\pi}{9}}{1-\tan\frac{\pi}{18}\tan\frac{\pi}{9}}$

Homework 2/26

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