Warm Up 2/9

Are the following equalities true?

a) $2 - 9(3 + 5) = -(100 - 65 + 21)$

$2 - 27 - 45$

$-70 \neq -56$ F

b) $3x - 4 \neq 3(x - 1)$

$3x - 3$

Verify the following equalities.

c) $x + 3 = 6\left(\frac{1}{6}x + \frac{11}{6} - \frac{13}{6} + \frac{5}{6}\right)$

$= \cancel{6}(\cancel{1/6}x + 3/\cancel{6})$

$= x + 3$

T

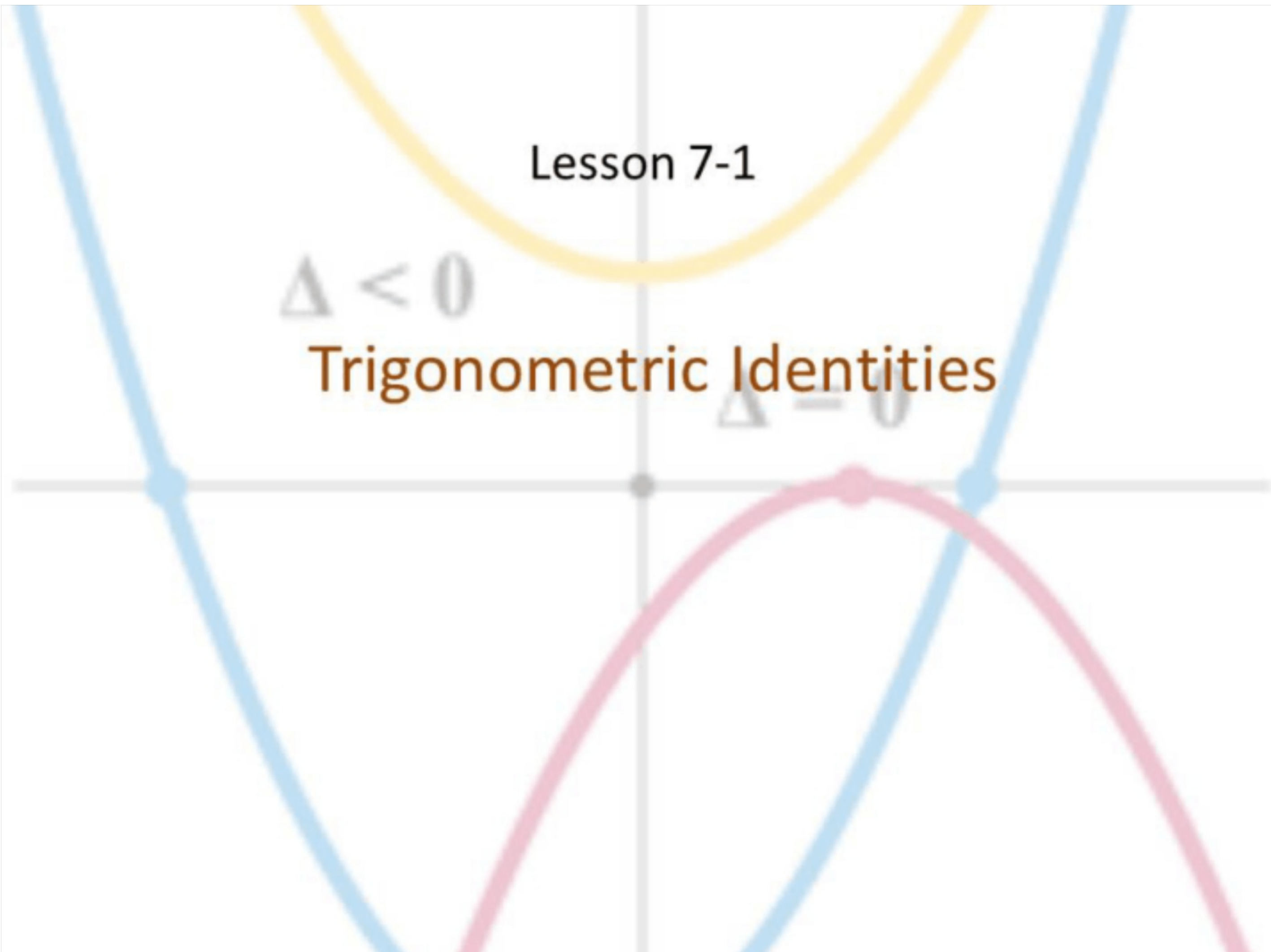
d) $3(2x - 1) = 2x + \cancel{2}\left(2x - \frac{3}{\cancel{2}}\right)$

$2x + 4x - 3$

$= 6x - 3$

$3(2x - 1)$

T


Lesson 7-1
Δ < 0
Trigonometric Identities

Objective

Students will…

- Be able to know the difference between an identity and an equation.
- Be able to know and apply the fundamental trigonometric identities to rewrite and simplify trigonometric expressions.

Analytic Trigonometry

In this chapter, we begin to break down and see how the trigonometric functions relate to one another, and take a closer look at what they mean **analytically**.

To analyze means to **separate into its parts** or elements, and **think critically** in order to bring out the essential elements.

id: $\overbrace{2x+x}=3x$
$3x=3x$

First, we need to define what an identity is. **Identity** is a mathematical expression written in a different way. An **equation** is a set of **simplified** expressions that are equal.

$\frac{4x}{2}=\frac{8x}{4}$

$y=x$ eq.

One way to differentiate between an equation and an identity is that if one side of an equation can be simplified or computed to equal another side, it is an identity.

Example

Classify the following as an identity or an equation.

id / eq $2x + 2 = 2x + 2$

id $2x + x = 3x$

eq $x + 1 = 2x + 2$

$-1 \qquad -1$

id $2(x + 1) = 2x + 2$

id $\tan x = \frac{\sin x}{\cos x}$

eq $\tan x = 3x - 9$

Trigonometric Identities

Before we get any deep into trig analysis, we must first recall some of the basic trigonometric identities and definitions. Primarily,

$$\csc x = \frac{1}{\sin x} \qquad \sec x = \frac{1}{\cos x} \qquad \cot x = \frac{1}{\tan x}$$

$$\tan x = \frac{\sin x}{\cos x} \qquad \cot x = \frac{\cos x}{\sin x}$$

Pythagorean Identity: $\sin^2 x + \cos^2 x = 1$

From this, we also get:

$1 - \sin^2 x = \cos^2 x$

$1 - \cos^2 x = \sin$

$$\tan^2 x + 1 = \sec^2 x \quad \text{and} \quad 1 + \cot^2 x = \csc^2 x$$

Pythagorean Identities

Let us now take the time how we get the other two Pythagorean identities from: $\sin^2 x + \cos^2 x = 1$

$\tan^2 x + 1 = \sec^2 x$

$$\frac{\sin^2 x}{\cos^2 x} + \frac{\cos^2 x}{\cos^2 x} = \frac{1}{\cos^2 x}$$

$$= \tan^2 x + 1 = \sec^2 x$$

$1 + \cot^2 x = \csc^2 x$

$$\frac{\sin^2 x}{\sin^2 x} + \frac{\cos^2 x}{\sin^2 x} = \frac{1}{\sin^2 x}$$

$$1 + \cot^2 x = \csc^2 x$$

Even-Odd Identities

We must recall the idea of the even-odd functions. Remember that cosine is an **even** function, while sine is an **odd** function.

Even:

$\cos(-x) = \cos x$ $\qquad\qquad$ $\sec(-x) = \sec x$

Odd:

$\sin(-x) = -\sin x$ $\qquad\qquad$ $\csc(-x) = -\csc x$

$\tan(-x) = -\tan x$ $\qquad\qquad$ $\cot(-x) = -\cot x$

Simplifying Trigonometric Expressions

We can use the identities and definitions to simply trig expressions. Let us simplify the following expression:

$$\cos t + \tan t \sin t$$

$$= \cos t + \frac{\sin t}{\cos t}(\sin t).$$

$$= \frac{\cos t \cdot \cos t}{\cos t \cdot 1} + \frac{\sin^2 t}{\cos t} \Rightarrow \frac{\cos^2 t}{\cos t} + \frac{\sin^2 t}{\cos t}$$

$$= \frac{\cos^2 t + \sin^2 t}{\cos t} = \frac{1}{\cos t}$$

$$= \sec t$$

ex. $1/3 + 1/5$ $\quad$ $1/6 + 1/8$

Example

$\cos\theta$

Simplify the expression:

$$\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{1+\sin\theta} = \cos\theta + \sin\theta\tan\theta$$

$$\frac{\sin\theta + \sin^2\theta + \cos^2\theta}{\cos\theta(1+\sin\theta)} = \frac{\sin\theta + 1}{\cos\theta\cancel{(1+\sin\theta)}} = \frac{1}{\cos\theta}$$

$$= \sec\theta$$

Example

Simplify the expression:

$$(1 + \sin y)(\sec y - \tan y)$$

$$(1+\sin y)\left(\frac{1}{\cos y} - \frac{\sin y}{\cos y}\right)$$

$$= (1+\sin y)\left(\frac{1-\sin y}{\cos y}\right)$$

$$= \frac{1-\cancel{\sin y}}{\cos y} + \frac{\cancel{\sin y} - \sin^2 y}{\cos y}$$

$$= \frac{1-\sin^2 y}{\cos y} = \frac{\cos^2 y}{\cos y}$$

$$= \boxed{\cos y}$$

Example

Simplify the expression:

$$\frac{\sin\theta + \cot\theta\cos\theta}{\cot\theta} = \frac{\sin\theta + \frac{\cos\theta}{\sin\theta}\cos\theta}{\frac{\cos\theta}{\sin\theta}}$$

$$= \frac{\frac{\sin\theta}{1} + \frac{\cos^2}{\sin\theta}}{\frac{\cos\theta}{\sin\theta}} = \frac{\frac{\sin^2\theta + \cos^2\theta}{\sin\theta}}{\frac{\cos\theta}{\sin\theta}}$$

$$= \frac{1}{\cancel{\sin\theta}} \cdot \frac{\cancel{\sin\theta}}{\cos\theta} = \frac{1}{\cos\theta} = \sec\theta$$

Homework Problems

Write the trig expression in terms of sine and cosine, and then simplify:

1. $\tan^2 x - \sec^2 x$

Homework Problems

Simplify the expression:

11. $\frac{\sin x \sec x}{\tan x}$

Homework Problems

Simplify the expression:

15. $\frac{\sec^2 x - 1}{\sec^2 x}$

Homework Problems

Simplify the expression:

21. $\frac{2+\tan^2 x}{\sec^2 x} - 1$

Homework 2/9

TB pg. 533 #1, 5, 9, 11-23 (odd)