

Objective

Students will...

- Be able evaluate the integrals of exponential functions.
- Be able to evaluate the integrals of logarithmic functions.

Natural Exponential Functions

Recall, the number e. We define the natural exponential function as $f(x) = e^x$, where $e \approx 2.718281828459...$

The derivative of the natural exponential function is a simple chain rule:

$$\frac{d}{dx}[e^u] = e^u(u')$$

In other words, the derivative of e^u is e^u times the derivative of u.

Natural Exponential Functions

That being said, then, the integral of natural exponential functions is also quite simple:

Let u be a differentiable function of x.

$$1. \int e^x dx = e^x + C$$

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$$2. \int e^u du = e^u + C$$

In other words, it's a simple "U"-Substitution.

Example

Find
$$\int e^{3x+1} dx$$
 $|u-3x+1|$
 $|u-3x+1|$
 $|u-3x+1|$
 $|u-3|$
 $|u-3|$

Example

Find
$$\int \frac{e^{\frac{1}{x^2}}}{x^2} dx = \int e^{x} x^{-2} dx$$

$$U = x - \frac{1}{2}$$

$$du = -x^2 dx$$

$$-du = x^{-2} dx$$

$$= -e^{x} + C$$

$$-e^{x} + C$$

$$= -e^{x} + C$$

Example

Find
$$\int \sin x e^{\cos x} dx = -\int e^{u} du = -\left(e^{u} + C\right)$$
 $u = \cos x$
 $du = -\sin x dx$
 $du = \sin x dx$

Natural Logarithmic Functions

Recall that the inverse function of e^x is known as the natural logarithmic function, or namely, $f(x) = \ln x$.

Remember,
$$b = e^a \rightarrow \ln b = a$$

The derivative of the natural log function is also a chain rule as follows:

$$\frac{d}{dx}[\ln u] = \frac{1}{u}(u') = \frac{u'}{u}$$

So the derivative of $\ln u$ is 1 over u times the derivative of u.

$Q \cap (-\alpha) = Q \cap \mathcal{E}_{\mathbf{N}}$ Natural Logarithmic Functions

Then, the integration rule is as follows:

Let u be a differentiable function of x.

1.
$$\int \frac{1}{x} dx = \ln|x| + C$$
 2. $\int \frac{1}{u} du = \ln|u| + C$

$$2. \int \frac{1}{u} du = \ln|u| + C$$

$$3. \int \frac{u'}{u} dx = \ln|u| + C$$

It is also an application of the "U"-Substitution

Find
$$\int_{x}^{2} dx$$

Find $\int_{x}^{2} dx$

Find $\int_$

$$\int \frac{\mathcal{V}}{\mathcal{U}} \qquad \qquad \text{Example}$$
Find
$$\int \frac{3x^2 + 1}{x^3 + x} dx = \left(\frac{1}{x^3 + x} \right) \frac{1}{x^3 + x} dx$$

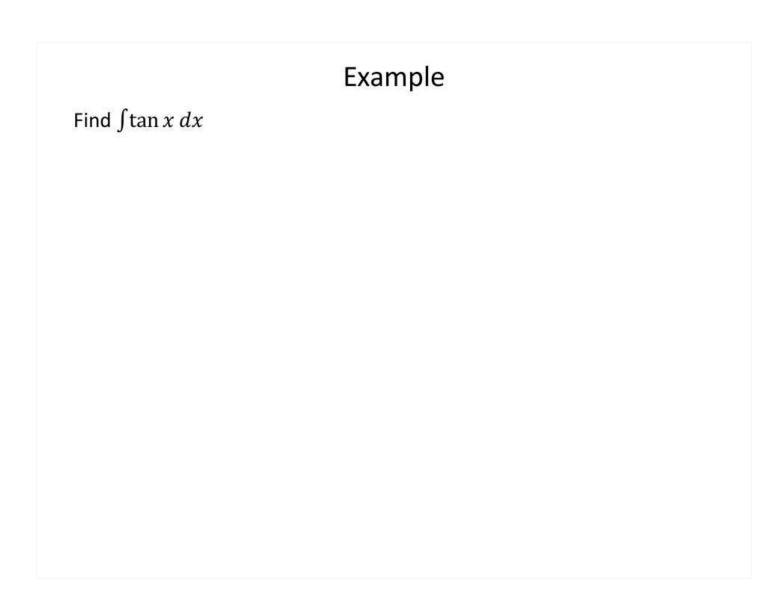


Find
$$\int \frac{x+1}{x^2+2x} dx = \frac{1}{2} \left(\frac{x^2+7x}{x^2+2x} + C \right)$$

Find
$$\int \frac{2x}{(x+1)^2} dx$$

$$U = \chi + \sqrt{3} \chi = (1-1)$$

$$|U| = | d\chi$$



Integrals of Six Basic Trig Functions

INTEGRALS OF THE SIX BASIC TRIGONOMETRIC FUNCTIONS

$$\int \sin u \, du = -\cos u + C \qquad \int \cos u \, du = \sin u + C$$

$$\int \tan u \, du = -\ln|\cos u| + C \qquad \int \cot u \, du = \ln|\sin u| + C$$

$$\int \sec u \, du = \ln|\sec u + \tan u| + C \qquad \int \csc u \, du = -\ln|\csc u + \cot u| + C$$

Homework 2/6

5.1 #1-23 (e.o.o), 25-35 (e.o.o), 47, 51 5.4 #85-105 (odd)