Warm Up 1/29

Use the Law of Sines to solve the triangle.

$a = 26, c = 15, \angle C = 29°$

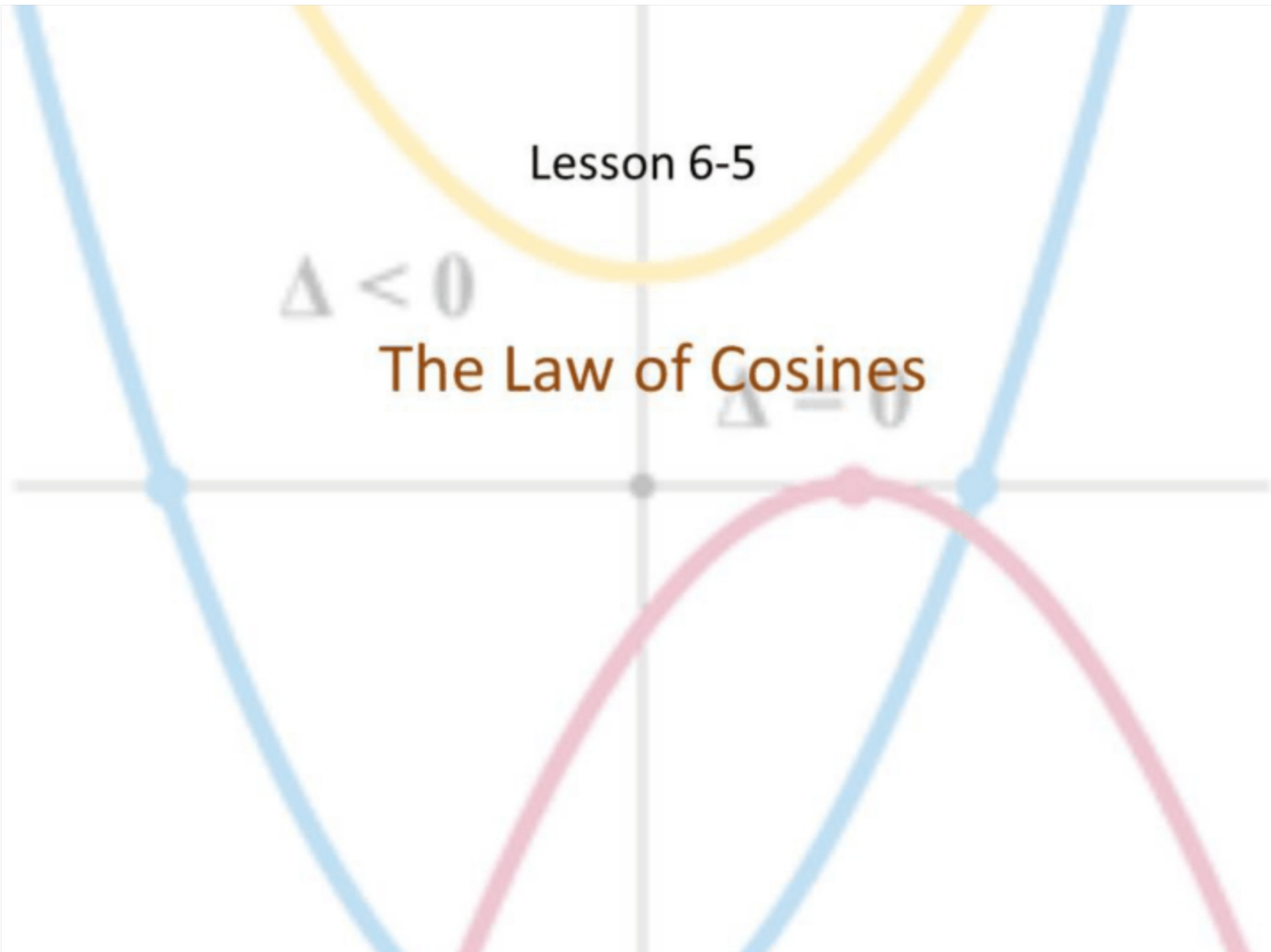
$$\frac{\sin 29}{15} = \frac{\sin A}{26} \Rightarrow A_1 \approx 57 \quad A_2 \approx 123$$

$B_1 \approx 94°$

$B_2 \approx 28°$

$b_1 \approx 31$

$b_2 \approx 15$


Lesson 6-5
The Law of Cosines
Δ < 0

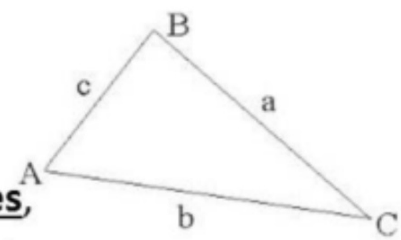
Objective

Students will…

- Be able to know what Law of Cosines is.
- Be able to apply the Law of Cosines to solve for missing sides or angles.

Triangles

We've been studying the trigonometric ratios involving right triangles. Trigonometry can also be used for **non**-right triangles. First thing we need to do is to be consistent with our notations.

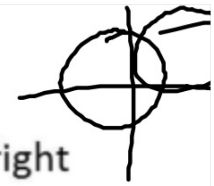


Consider the triangle ΔABC shown on the right.

The uppercase letters A, B, C represent the **vertices**, or the **angles** of the triangle, while the lower case letters a, b, c represent the sides. For ease, the angles will always be labeled by uppercase letters, while the side **opposite** to each angle, will always be labeled with the lowercase letter of the opposite angle.

So, from our picture, we see that a is the side opposite to A, while b is the side opposite to B and c is the side opposite to C.

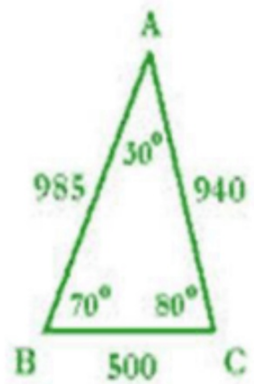
Law of Cosines



There exists another important law regarding triangles (not just right triangles).

Law of Cosines- In any triangle, say, ΔABC, we have:

$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$b^2 = a^2 + c^2 - 2ac\cos B$$
$$c^2 = a^2 + b^2 - 2ab\cos C$$



For the ΔABC to the left, we have...

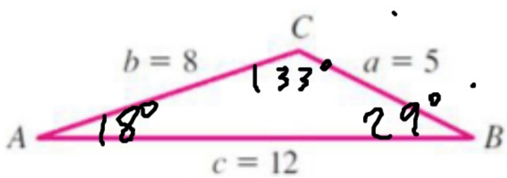
$$940^2 = 500^2 + 985^2 - 2(500)(985)\cos 70^\circ$$

Example

So we can apply the Law of Cosines to solve for missing sides or angles.

(Important: Make sure your calculator is in the right mode!)

Find the angles of the triangle.



$c^2 = b^2 + a^2 - 2ba\cos C$

$144 = 64 + 25 - 2(8)(5)\cos C$

$144 = 89 - 80\cos C$

$-89 \quad -89$

$55 = -80\cos C \Rightarrow -\frac{55}{80} = \cos C$

$C \approx 133^\circ$

$b^2 = a^2 + c^2 - 2ac\cos B$

$64 = 25 + 144 - 2(5)(12)\cos B$

$-105 = -120\cos B$

$\cos B = \frac{105}{120}$

$B = 29^\circ$

Example

Solve ΔABC, where $\angle A = 46.5°$, $b = 10.5$, and $c = 18$

$B \approx 48.5°$

$a^2 = b^2 + c^2 - 2bc\cos A$

$a^2 = (10.5)^2 + 18^2 - 2(10.5)(18)\cos 46.5$

$434.25 - 378\cos 46.5$

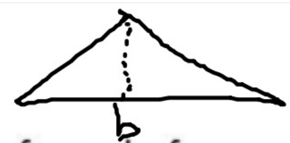
$\Rightarrow a \approx 13$

$18^2 = 13^2 + 10.5^2 - 2(13)(10.5)\cos C$

$-23.25 = -273\cos C \Rightarrow \cos C = \frac{+23.75}{+273}$

$C \approx 85$

Heron's (Area) Formula



An interesting application of the Law of Cosines involves a formula for finding the area of a triangle from the lengths of its three sides. We won't derive the formula here for time's sake. (see textbook)

<u>Heron's Formula</u>- For ΔABC the area $\mathcal{A} = \sqrt{s(s-a)(s-b)(s-c)}$, where $s = \frac{1}{2}(a+b+c)$, which is the **<u>semiperimeter</u>** (half perimeter).

Ex. Find the area of a triangle with given side lengths:
$a = 280$, $b = 125$, and $c = 315$

$S = \frac{280+125+315}{2} = 360$

$A = \sqrt{360(360-280)(360-125)(360-315)}$

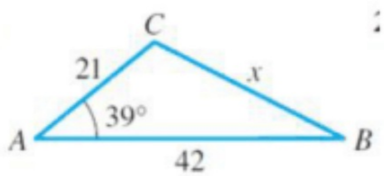
$= \sqrt{360(80)(235)(45)}$

≈ 17451.6

Homework Problems

Use the Law of Cosines to determine the indicated side x or angle θ.

1.



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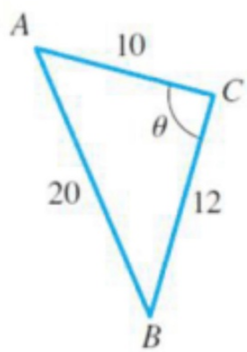

Homework Problems

Solve the triangle.

11. $a = 3, b = 4, \angle C = 53°$

Homework Problems

Solve the triangle.

17. $a = 50, b = 65, \angle A = 55°$

Homework Problems

Find the area of the triangle.

27. $a = 9, b = 12, c = 15$

Homework 1/29

TB pg. 513 #1, 3, 5, 8, 11-17 (odd), 27, 29