Warm Up 1/29

Use the Law of Sines to solve the triangle.

$a = 26, c = 15, \angle C = 29°$

$$\frac{\sin C}{c} = \frac{\sin A}{a}$$

$$\frac{\sin 29}{15} = \frac{\sin A}{26}$$

$A_1 \approx 57°$ $A_2 \approx 123°$

$B_1 \approx 94°$ $B_2 \approx 28°$

$b_1 \approx 31$ $b_2 \approx 15$

$$\frac{c}{\sin C} = \frac{b_1}{\sin B_1} \Rightarrow \frac{15}{\sin 29} = \frac{b_1}{\sin 94}$$

$$\Rightarrow b_1 \approx 31$$

$$\frac{c}{\sin C} = \frac{b_2}{\sin b_2}$$

$$\Rightarrow b_2 \approx 15$$

Lesson 6-5

The Law of Cosines

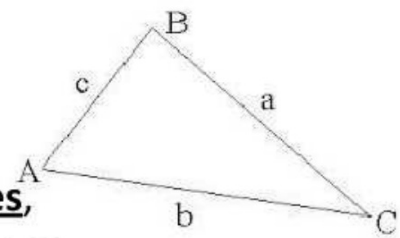
Objective

Students will…

- Be able to know what Law of Cosines is.
- Be able to apply the Law of Cosines to solve for missing sides or angles.

Triangles

We've been studying the trigonometric ratios involving right triangles. Trigonometry can also be used for **non**-right triangles. First thing we need to do is to be consistent with our notations.



Consider the triangle ΔABC shown on the right.

The uppercase letters A, B, C represent the **<u>vertices</u>**,

or the **<u>angles</u>** of the triangle, while the lower case letters

a, b, c represent the sides. For ease, the angles will always be labeled by uppercase letters, while the side **<u>opposite</u>** to each angle, will always be labeled with the lowercase letter of the opposite angle.

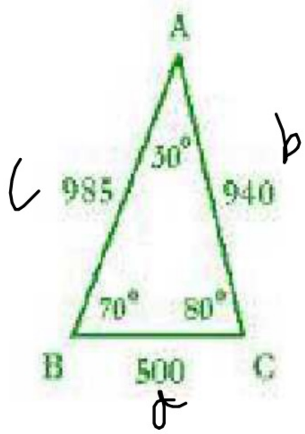
So, from our picture, we see that a is the side opposite to A, while b is the side opposite to B and c is the side opposite to C.

Law of Cosines

There exists another important law regarding triangles (not just right triangles).

Law of Cosines- In any triangle, say, ΔABC, we have:

$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$b^2 = a^2 + c^2 - 2ac\cos B$$
$$c^2 = a^2 + b^2 - 2ab\cos C$$



For the ΔABC to the left, we have...

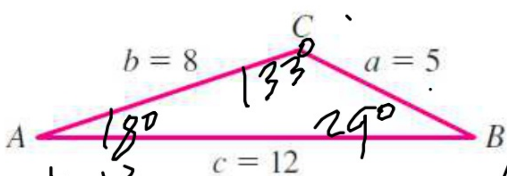
$$500^2 = 940^2 + 985^2 - 2(940)(985)\cos 30$$

Example

So we can apply the Law of Cosines to solve for missing sides or angles.

(Important: Make sure your calculator is in the right mode!)

Find the angles of the triangle.



$c^2 = a^2 + b^2 - 2ab\cos C$

$144 = 25 + 64 - 2(5)(8)\cos C$

$55 = -80\cos C$

$\frac{55}{-80} = \frac{-80\cos C}{-80}$

$a^2 = b^2 + c^2 - 2bc\cos A$

$5^2 = 8^2 + 12^2 - 2(8)(12)\cos A$

$25 = 64 + 144 - 192\cos A$

$25 = 208 - 192\cos A$

$-208 \quad -208$

$-183 = -192\cos A$

$\frac{-183}{-192} = \frac{-192\cos A}{-192}$

$18^\circ \approx A$

Example

Solve ΔABC, where $\angle A = 46.5°$, $b = 10.5$, and $c = 18$

$a^2 = b^2 + c^2 - 2bc\cos A$

$\sqrt{a^2} = \sqrt{10.5^2 + 18^2 - 2(10.5)(18)\cos 46.5}$

$a \approx 13.2$

$b^2 = a^2 + c^2 - 2ac\cos B$

$10.5^2 = 13.2^2 + 18^2 - 2(13.2)(18)\cos B$

$B \approx 35.3°$

$C \approx 98.2°$

Heron's (Area) Formula

An interesting application of the Law of Cosines involves a formula for finding the area of a triangle from the lengths of its three sides. We won't derive the formula here for time's sake. (see textbook)

<u>Heron's Formula</u>- For ΔABC the area $\mathcal{A} = \sqrt{s(s-a)(s-b)(s-c)}$, where $s = \frac{1}{2}(a+b+c)$, which is the **<u>semiperimeter</u>** (half perimeter).

Ex. Find the area of a triangle with give side lengths:
$a = 280$, $b = 125$, and $c = 315$

$s = \frac{280+125+315}{2} = \frac{720}{2} = 360$

$A = \sqrt{360(360-280)(360-125)(360-315)}$

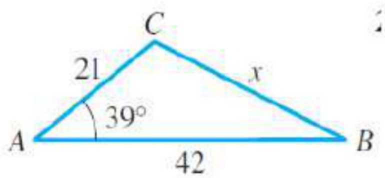
$= \sqrt{360(80)(235)(45)}$

≈ 17452

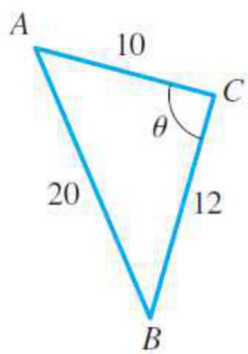
Homework Problems

Use the Law of Cosines to determine the indicated side x or angle θ.

1.



8.



Homework Problems

Solve the triangle.

11. a =3, $b = 4$, $\angle C = 53°$

Homework Problems

Solve the triangle.

17. $a = 50, b = 65, \angle A = 55°$

Homework Problems

Find the area of the triangle.

27. $a = 9, b = 12, c = 15$

Homework 1/29

TB pg. 513 #1, 3, 5, 8, 11-17 (odd), 27, 29