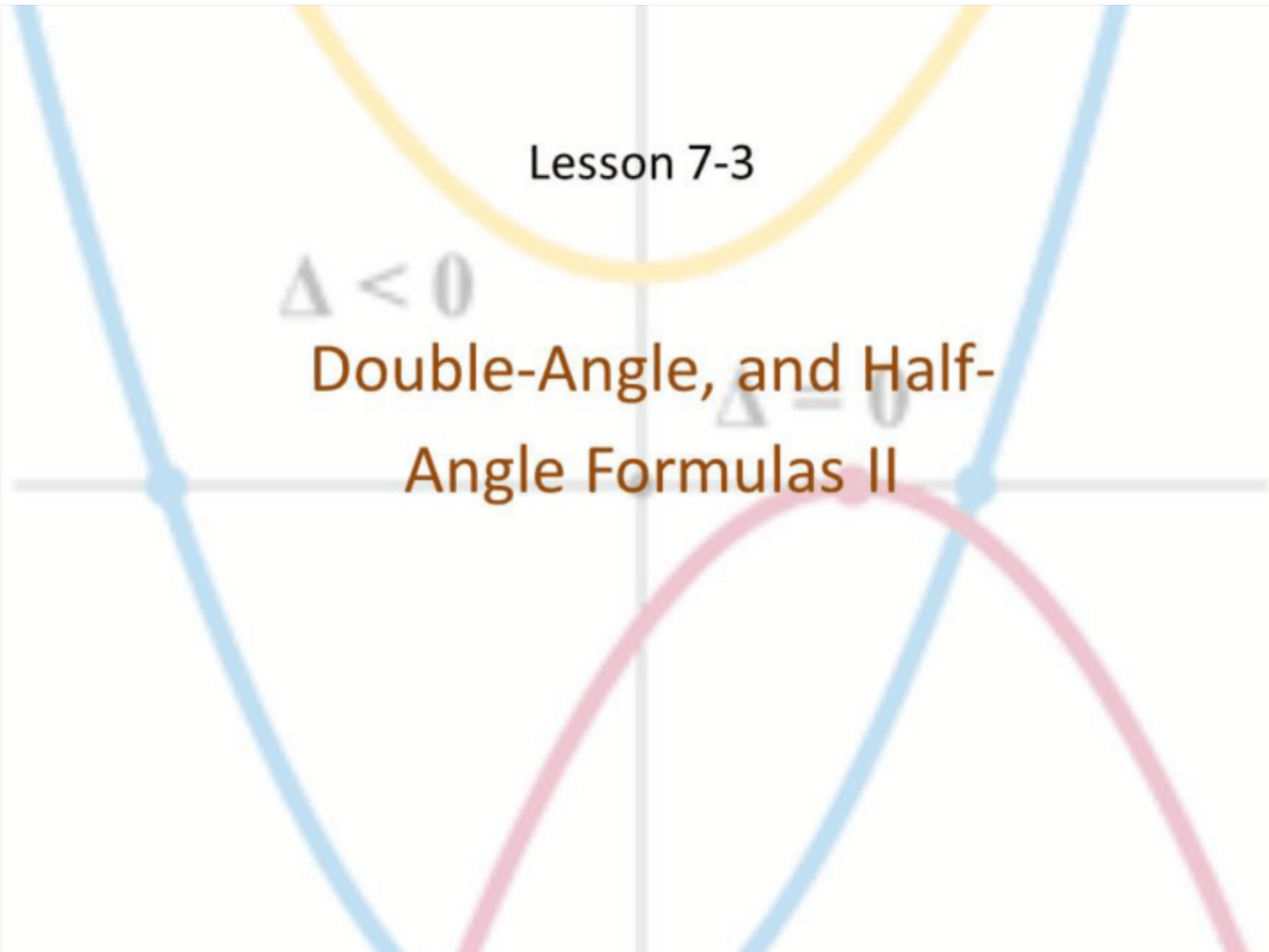
Lesson 7-3

Double-Angle, and Half-Angle Formulas II



Objective

Students will…

- Be able to apply the Double-Angle and Half-Angle formulas to verify identities.

Double-Angle Formulas

The following formulas are direct results of addition and subtraction formulas. **Double-Angle** formulas allows us to find the values of the trigonometric functions at $2x$ from their values at x.

Double-Angle Formulas:

For Sine: $\sin(2x) = 2\sin x \cos x$

$\sin(4x) = 2\sin 2x \cos 2x$

$\sin(12x) = 2\sin 6x \cos 6x$

$\sin(14x) = 2\sin 7x \cos 7x$

For Cosine: $\cos(2x) = \cos^2 x - \sin^2 x$

$= 1 - 2\sin^2 x$

$= 2\cos^2 x - 1$

$\cos(6x) = \cos^2 3x - \sin^2 3x$

For Tangent: $\tan(2x) = \frac{2\tan x}{1 - \tan^2 x}$

Proof of Double-Angle Formulas

Prove the formula: $\cos(2x) = \cos^2 x - \sin^2 x$

$$\cos(x+x) = \cos x \cos x - \sin x \sin x$$
$$= \cos^2 x - \sin^2 x.$$

Show that $\cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$

$$\cos^2 x - \sin^2 x = \cos^2 x - (1 - \cos^2 x) = 2\cos^2 x - 1$$
$$\cos^2 x - \sin^2 x = 1 - \sin^2 x - \sin^2 x = 1 - 2\sin^2 x$$

Proof of Double-Angle Formulas

Prove the formula: $\sin(2x) = 2\sin x \cos x$

$\sin(x+x) = \sin x \cos x + \cos x \sin x = 2\sin x \cos x$

Prove the formula: $\tan(2x) = \frac{2\tan x}{1-\tan^2 x}$

$$\tan(x+x) = \frac{\tan x + \tan x}{1 - \tan x \tan x} = \frac{2\tan x}{1-\tan^2 x}$$

Half-Angle Formulas

The next set of formulas relate the values of trig functions at $\frac{1}{2}x$ to their values at x. They are known as the **Half-Angle Formulas**.

Half-Angle Formulas:

$$\sin\frac{u}{2} = \pm\sqrt{\frac{1-\cos u}{2}} \qquad \cos\frac{u}{2} = \pm\sqrt{\frac{1+\cos u}{2}}$$

$$\tan\frac{u}{2} = \frac{1-\cos u}{\sin u} = \frac{\sin u}{1+\cos u}$$

*The choice of $+$ or $-$ depends on which quadrant $\frac{u}{2}$ lies in.

Guidelines for Proving Identities

1. Always look for opportunities to use the Sum-to-Product formulas, before applying the Double-Angle formulas.

2. When either can be used, it's **usually** best to use the Double-Angle formulas, rather than the Half-Angle formulas. Half-Angle formulas are rarely used when proving identities.

3. With regards to the Double-Angle formulas, always look for multiples of 2. When whole numbers are doubled, they are always even (i.e. multiples of 2!).

Example

Prove the identity: $\frac{\sin 3x}{\sin x \cos x} = 4\cos x - \sec x$

$$LHS: \frac{\sin(x+2x)}{\sin x \cos x} = \frac{\sin x \cos 2x + \cos x \sin 2x}{\sin x \cos x}$$

$$= \frac{\cancel{\sin x}\cos 2x}{\cancel{\sin x}\cos x} + \frac{\cancel{\cos x}\sin 2x}{\sin x \cancel{\cos x}} = \frac{2\cos^2 x - 1}{\cos x} + \frac{2\cancel{\sin x}\cos}{\cancel{\sin x}}$$

$$= \frac{2\cos^{\cancel{2}} x}{\cancel{\cos x}} - \frac{1}{\cos x} + 2\cos x = 4\cos x - \sec x = RHS \checkmark$$

Homework Problems

Prove the identity.

60. $\sin 8x = 2\sin 4x \cos 4x$

LHS: $2\sin 4x \cos 4x$ = RHS ✓

Homework Problems

Prove the identity.

62. $\frac{2\tan x}{1+\tan^2 x} = \sin 2x$

LHS: $\frac{2\tan x}{\sec^2 x} = 2\tan x \cos^2 x = 2\,\frac{\sin x}{\cancel{\cos x}}\,\frac{\cos^{\cancel{2}} x}{1}$

$= 2\sin x\cos x$

$= \sin 2x$ = RHS

Homework Problems

Prove the identity.

64. $\frac{1+\sin 2x}{\sin 2x} = 1 + \frac{1}{2}\sec x \csc x$

$$\frac{1}{\sin 2x} + \cancel{\frac{\sin 2x}{\sin 2x}}\ 1$$

$$= \frac{1}{2\sin x\cos x} + 1$$

$$= 1 + \frac{1}{2}\sec x \csc x$$

$= RHS$ ✓

Homework Problems

66. $\cot 2x = \frac{1-\tan^2 x}{2\tan x}$

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