

$$(x+y)(x-y) = x^2 - y^2$$

Warm Up 3/4

Verify the identity.

$$\cos(x+y)\cos(x-y) = \cos^2 x - \sin^2 y$$

$$\text{LHS} = (\cos x \cos y - \sin x \sin y)(\cos x \cos y + \sin x \sin y)$$

$$= \cos^2 x \cos^2 y - \sin^2 x \sin^2 y$$

$$= \cos^2 x (1 - \sin^2 y) - \sin^2 y (1 - \cos^2 x)$$

$$= \cos^2 x - \sin^2 y \cos^2 x - \sin^2 y + \sin^2 y \cos^2 x$$

$$= \cos^2 x - \sin^2 y = \text{RHS}$$

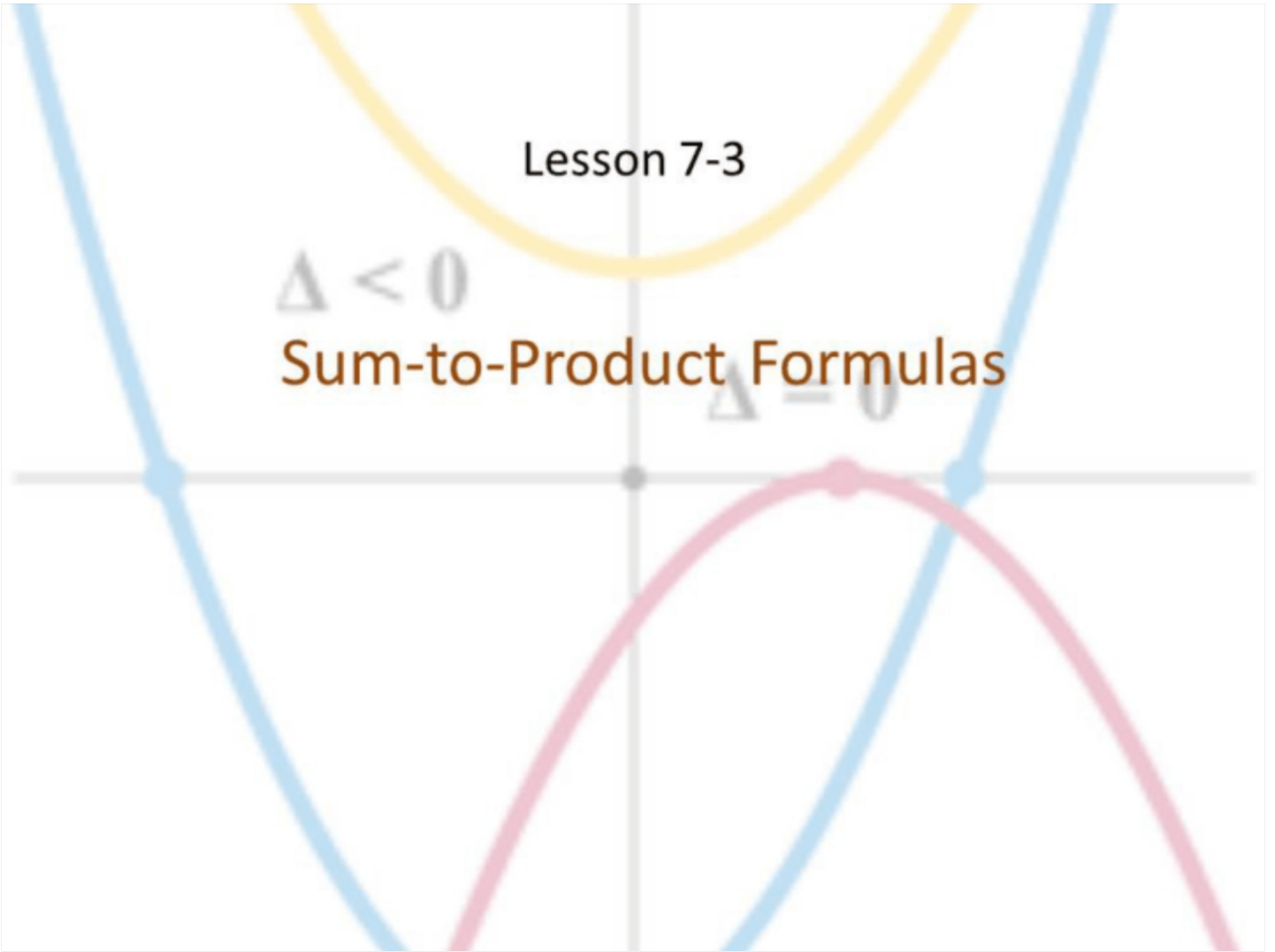


Lesson 7-3

$\Delta < 0$

Sum-to-Product Formulas

$\Delta = 0$



Objective

Students will...

- Be able to know the Sum-to-Product Formulas.
- Be able to use the Sum-to-Product formulas to prove identities.

+ -

Sum-to-Product Formulas

We now move further into different formulas for trig functions. The following is the **Sum-to-Product** formulas, which do exactly as ~~it~~ ^{the} says turn sums (addition or subtraction) into products (multiplication).

Sum-to-Product Formulas:

$$\sin x + \sin y = 2 \sin \frac{x+y}{2} \cos \frac{x-y}{2}$$

$$\sin x - \sin y = 2 \cos \frac{x+y}{2} \sin \frac{x-y}{2}$$

$$\cos x + \cos y = 2 \cos \frac{x+y}{2} \cos \frac{x-y}{2}$$

$$\cos x - \cos y = 2 \sin \frac{x+y}{2} \sin \frac{x-y}{2}$$

Using Sum-to-Product Formulas

Write $\sin 7x + \sin 3x$ as a product.

$$= 2 \sin \frac{7x+3x}{2} \cos \frac{7x-3x}{2} = 2 \sin \frac{10x}{2} \cos \frac{4x}{2}$$

$$= 2 \sin 5x \cos 2x$$

Write $\sin 11x + \sin 5x$ as a product.

$$= 2 \sin 8x \cos 3x$$

$$2 \sin \frac{11x+5x}{2} \cos \frac{11x-5x}{2} = 2 \sin \frac{16x}{2} \cos \frac{6x}{2} =$$

Homework Problem

Write the sum as a product.

$$49. \cos 4x - \cos 6x$$

$$= -2 \sin \frac{4x+6x}{2} \sin \frac{4x-6x}{2} = -2 \sin(5x) \sin(-x)$$

$$= \boxed{2 \sin 5x \sin x}$$

~~f(x)~~
~~g(x)~~

Using Sum-to-Product Formulas

Verify the identity: $\frac{\sin 3x - \sin x}{\cos 3x + \cos x} = \tan x$

$$\text{LHS} = \frac{\cancel{2} \cos \frac{3x+x}{2} \sin \frac{3x-x}{2}}{\cancel{2} \cos \frac{3x+x}{2} \cos \frac{3x-x}{2}}$$

$$\therefore = \frac{\sin\left(\frac{2x}{2}\right)}{\cos\left(\frac{2x}{2}\right)} = \frac{\sin x}{\cos x} = \tan x = \text{RHS}$$



Using Sum-to-Product Formulas

Verify the identity: $\frac{\sin 4x + \sin 2x}{\sin 2x} = \frac{\sin 3x}{\sin x}$

Homework Problems

Verify the identity: 71. $\frac{\sin x + \sin 5x}{\cos x + \cos 5x} = \tan 3x$

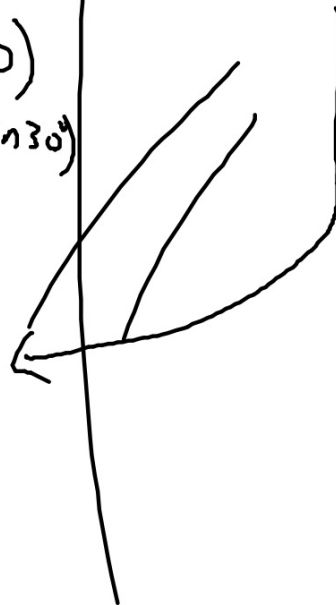
Homework Problems

$\sin(x+y)$

79. Show that $\sin 45^\circ + \sin 15^\circ = \sin 75^\circ$

$$\begin{aligned}
 &= 2 \sin \frac{45^\circ + 15^\circ}{2} \cos \frac{45^\circ - 15^\circ}{2} \\
 &= 2 \sin 30^\circ \cos 15^\circ \\
 &= 2 \sin 30^\circ \cos(45^\circ - 30^\circ) \\
 &= 2 \sin 30^\circ (\cos 45^\circ \cos 30^\circ + \sin 45^\circ \sin 30^\circ) \\
 &= 2 \left(\frac{1}{2}\right) \left(\frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{1}{2}\right) \\
 &= \left(\frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{1}{2}\right)
 \end{aligned}$$

$$\begin{aligned}
 \sin 75^\circ &= \sin(45^\circ + 30^\circ) \\
 &= \sin 45^\circ \cos 30^\circ + \cos 45^\circ \sin 30^\circ \\
 &= \left(\frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{1}{2}\right)
 \end{aligned}$$



Homework 3/4

TB pg. 548-549 #47-53 (odd), 72, 74, 75, 77

$$\cos(-x) = \cos x.$$

$$72. \quad \frac{\sin 3x + \sin 7x}{\cos 3x - \cos 7x} = \cot 2x$$

$$\text{LHS} = \frac{2 \sin \frac{3x+7x}{2} \cos \frac{3x-7x}{2}}{-2 \sin \frac{3x+7x}{2} \sin \frac{3x-7x}{2}} = \frac{\cancel{2} \sin(5x) \cos(-2x)}{-\cancel{2} \sin(5x) \sin(-2x)}.$$

$$= \frac{\cos(2x)}{-\sin(-2x)} = \frac{\cos 2x}{\sin 2x} = \cot 2x = \text{RHS}$$

