Lesson 7-1

Area Between Two Curves

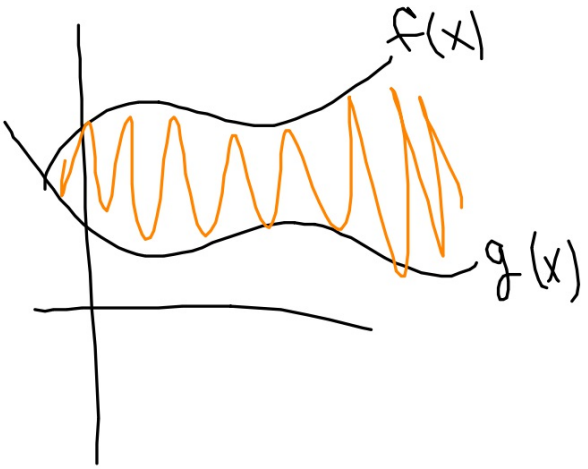
Objective

Students will…

- Be able to find the area of a region between two curves using integration.
- Be able to find the area of a region between intersecting curves using integration.

Area ~~Underneath~~ Between the Curve

One of the basic applications of integration deals with finding the area between two curves. Consider the following example.



$$\int_a^b f(x)\,dx - \int_a^b g(x)\,dx$$

upper

lower

Area Underneath the Curve

We can then formalize this process.

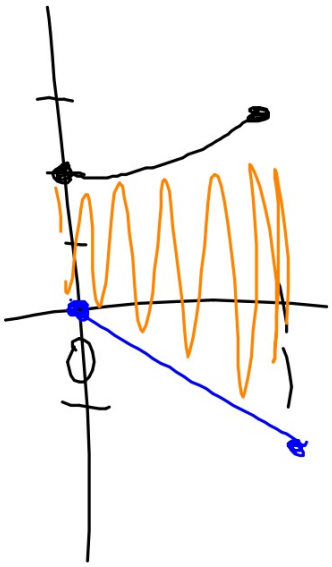
Area of a Region Between Two Curves- If f and g are continuous on $[a, b]$ and $g(x) \leq f(x)$ for all x in $[a, b]$, then the area of a the region bounded by the graphs of f and g and the vertical lines $x = a$ and $x = b$ is $A = \int_a^b [f(x) - g(x)]\, dx$

upper lower

② For horizontal orientation ($x = \ldots$) right − left

Examples

Find the area of the region bounded by the graphs of $y = x^2 + 2$, $y = -x$, $x = 0$, and $x = 1$.



$$A = \int_0^1 x^2 + 2 - (-x)\,dx$$

$$= \frac{1}{3}x^3 + 2x + \frac{1}{2}x^2 \Big|_0^1$$

$$\left(\frac{1}{3} + 2 + \frac{1}{2}\right) - 0 = \frac{17}{6}$$

Examples

Find the area of the region bounded by the graphs of $f(x) = 2 - x^2$ and $g(x) = x$.

$2 - x^2 = x$

$x^2 + x - 2 = 0$

$(x+2)(x-1) = 0$

$x = -2, 1$

0 is in $[-2, 1]$

$f(0) = 2$

$g(0) = 0 \Rightarrow f(x) > g(x)$

$$A = \int_{-2}^{1} 2 - x^2 - x \, dx = 2x - \frac{1}{3}x^3 - \frac{1}{2}x^2 \Big|_{-2}^{1}$$

$$\left(2 - \frac{1}{3} - \frac{1}{2}\right) - \left(-4 + \frac{8}{3} - 2\right)$$

$$= 8 - 3 - \frac{1}{2} = \boxed{\frac{9}{2}}$$

Examples

Find the area of the region bounded by the graphs of $f(x) = 3x^3 - x^2 - 10x$ and $g(x) = -x^2 + 2x$.

$3x^3 - x^2 - 10x = -x^2 + 2x$

$3x^3 - 12x = 0$

$3x(x^2 - 4) = 0$

$x = 0, \pm 2$

$$A = \int_{-2}^{0} 3x^3 - x^2 - 10x - (-x^2 + 2x)\,dx + \int_{0}^{2} -x^2 + 2x - (3x^3 - x^2 - 10x)\,dx$$

$$= \int_{-2}^{0} 3x^3 - 12x\,dx + \int_{0}^{2} -3x^3 + 12x\,dx$$

12

24

Examples

Find the area of the region bounded by the graphs of $x = 3 - y^2$ and $x = y + 1$.

right − left.

$y+1 = 3-y^2$

$y^2+y-2=0$

$(y+2)(y-1)=0$

$y=-2,1$

$$A=\int_{-2}^{1} 3-y^2-(y+1)\,dy=\int_{-2}^{1} -y^2-y+2\,dy$$

$$=-\frac{1}{3}y^3-\frac{1}{2}y^2+2y\Big|_{-2}^{1}$$

$$\left(-\frac{1}{3}-\frac{1}{2}+2\right)-\left(\frac{8}{3}-2-4\right)$$

$$\frac{9}{6}-\left(-\frac{18}{6}\right)=\frac{27}{6}=\frac{a}{2}$$

Homework 2/20

7.1 #1-6, 13, 14, 17-41 (e.o.o)