Warm Up 2/11

Simplify the trigonometric expression.

1. $\frac{\sec x - \cos x}{\tan x}$

Lesson 7-1

Trigonometric Identities II

Objective

Students will…

- Be able to prove or verify trigonometric identities.
- Be able to simplify expressions using trigonometric substitution.

Trigonometric Identities

Before we get any deep into trig analysis, we must first recall some of the basic trigonometric identities and definitions. Primarily,

$$\csc x = \frac{1}{\sin x} \qquad \sec x = \frac{1}{\cos x} \qquad \cot x = \frac{1}{\tan x}$$

$$\tan x = \frac{\sin x}{\cos x} \qquad \cot x = \frac{\cos x}{\sin x}$$

Pythagorean Identity: $\sin^2 x + \cos^2 x = 1$

From this, we also get:

$\sin^2 x = 1 - \cos^2 x$ and $\cos^2 x = 1 - \sin^2 x$

$\tan^2 x + 1 = \sec^2 x$ and $1 + \cot^2 x = \csc^2 x$

$\tan^2 x = \sec^2 x - 1$ and $\cot^2 x = \csc^2 x - 1$

Methods for Proving Identities

One of the main components of trig analysis is to prove identities. There are two different methods for proving identities:

I. Rewrite **one** of the sides to match the other side.

Ex.

$$x + 3 = 6\left(\frac{1}{6}x + \frac{11}{6} - \frac{13}{6} + \frac{5}{6}\right)$$

$= x + 11 - 13 + 5$

LHS $= x + 3$ ✓

II. Modify **both** sides until they are the same.

Ex.

$$3(2x - 1) = 2x + 2\left(2x - \frac{3}{2}\right)$$

$6x - 3$ $= 2x + 4x - 3$

$= 6x - 3$ ✓

Guidelines for Proving Identities

Furthermore, we have some guidelines/tips for proving identities.

1. **Focus on the fractions**: More often than not, identity proofs are more easily done when you work with the side that involves a fraction.

2. **Pick the more "complicated" side**: It's easier to modify the sides that has less sines or cosines. Generally, rewriting everything as sine or cosine can help you when you are "stuck."

3. **Use the Known Identities!**: Use algebra and the identities are already known to you. Look to combine multiple fractions into one with a common denominator.

Example

Prove/Verify the identity: $\cos\theta\,(\sec\theta - \cos\theta) = \sin^2\theta$

LHS: $\cos\theta(\sec\theta - \cos\theta)$

$= \cos\theta \cdot \frac{1}{\cos\theta} - \cos^2\theta$

$= 1 - \cos^2\theta$

$= \sin^2\theta = \text{RHS}$ ✓

Example

Prove/Verify the identity: $\cos x \tan x = \sin x$

LHS: $\cos x \tan x$

$= \cancel{\cos x} \dfrac{\sin x}{\cancel{\cos x}}$

$= \sin x = RHS$

Homework Problems

Verify the identity:

31. $\sin B + \cos B \cot B = \csc B$

Homework Problems

Verify the identity :

29. $\frac{\tan y}{\csc y} = \sec y - \cos y$

Homework Problems

Verify the identity :

35. $\tan\theta + \cot\theta = \sec\theta \csc\theta$

Homework Problems

Verify the identity :

51. $\frac{1-\cos\alpha}{\sin\alpha} = \frac{\sin\alpha}{1+\cos\alpha}$

Homework Problems

Verify the identity :

85. $\frac{1+\sin x}{1-\sin x} = (\tan x + \sec x)^2$

Homework 2/11

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