

$$37) \frac{\sin(x+y) - \sin(x-y)}{\cos(x+y) + \cos(x-y)} = \tan y.$$

$$\text{LHS: } \frac{\cancel{\sin x \cos y} + \sin y \cos x - \cancel{\sin x \cos y} + \sin y \cos x}{\cancel{\cos x \cos y} - \cancel{\sin x \sin y} + \cos x \cos y + \cancel{\sin x \sin y}}$$

$$= \frac{2 \sin y \cos x}{2 \cos x \cos y} = \tan y = \text{RHS } \checkmark$$

$$35) \tan x - \tan y = \frac{\sin(x-y)}{\cos x \cos y}$$

$$\text{RHS: } \frac{\sin x \cos y - \sin y \cos x}{\cos x \cos y}$$

$$= \frac{\cancel{\sin x \cos y}}{\cancel{\cos x \cos y}} - \frac{\cancel{\sin y \cos x}}{\cancel{\cos x \cos y}}$$

$$= \tan x - \tan y = \text{LHS} \quad \checkmark$$

$$\begin{aligned}
 & 19) \quad \tan\left(\frac{\pi}{2} - u\right) = \cot u \\
 \text{LHS: } & \frac{\tan \frac{\pi}{2} - \tan u}{1 + \tan \frac{\pi}{2} \tan u} = \frac{\frac{\cos u}{\sin u} - \frac{\cos u}{\sin u}}{1 + \frac{\cos u}{\sin u} \cdot \frac{\cos u}{\sin u}} = \frac{\frac{\cos u}{\sin u} - \frac{\cos u}{\sin u}}{\frac{\sin^2 u + \cos^2 u}{\sin^2 u}} \\
 & = \frac{\frac{\cos u}{\sin u} - \frac{\cos u}{\sin u}}{\frac{1}{\sin^2 u}} = \frac{\frac{\cos u}{\sin u} - \frac{\cos u}{\sin u}}{1} \\
 & = \frac{\cos u}{\sin u} = \cot u \\
 & = \text{RHS}
 \end{aligned}$$

$$\begin{aligned}
 34) \cot(x+y) &= \frac{\cot x \cot y - 1}{\cot x + \cot y} \\
 \text{HS} &= \frac{1}{\tan(x+y)} = \frac{1}{\frac{\tan x + \tan y}{1 - \tan x \tan y}} = \frac{1 - \tan x \tan y}{\tan x + \tan y} \cdot \frac{\cot x \cot y}{\cot x \cot y} = \frac{1 - \tan x \tan y}{\tan x + \tan y} \cdot \frac{1}{\cot x} \cdot \frac{1}{\cot y} \\
 &= \frac{1 - \tan x \tan y}{\tan x + \tan y} \cdot \frac{1}{\cot y \cot x} = \frac{1 - \tan x \tan y}{\tan x + \tan y} \cdot \frac{1}{\cot y \cot x} \\
 &= \frac{\cot x \cot y - 1}{\cot x \cot y} \\
 &= \frac{\cot y + \cot x}{\cot x \cot y}
 \end{aligned}$$

$$27) \tan(x - \pi) = \tan x$$

$$\text{LHS: } \frac{\tan x - \tan \pi}{1 + \tan x \tan \pi} = \frac{\tan x - 0}{1 + 0} = \tan x = \text{RHS} \checkmark$$

$$37) \frac{\sin(x+y) - \sin(x-y)}{\cos(x+y) + \cos(x-y)} = \tan y$$

$$\text{LHS: } \sin x \cos y + \sin y \cos x \quad \left(\overbrace{\sin x \cos y + \sin y \cos x} \right)$$

$$\frac{\cos x \cos y - \sin x \sin y + \cos x \cos y + \sin x \sin y}{\sin y \cos x} = \tan y = \text{RHS} \quad \checkmark$$

$$33) \quad \boxed{\cot(x-y) = \frac{\cot x \cot y + 1}{\cot y - \cot x}}$$

$$\text{LHS: } \frac{1}{\tan(x-y)} = \frac{1}{\frac{\tan x - \tan y}{1 + \tan x \tan y}} =$$

$$= \frac{\cot x \cot y + 1}{\cot x \cot y} = \frac{\cot x \cot y + 1}{\cancel{\cot x \cot y}}$$

$$\frac{1 + \tan x \tan y}{\tan x - \tan y} = \frac{1 + \frac{1}{\cot x} \frac{1}{\cot y}}{\frac{1}{\cot x} - \frac{1}{\cot y}} =$$

$$\frac{\cancel{\cot x \cot y} + 1}{\cot y - \cot x} = \frac{\cot x \cot y + 1}{\cot y - \cot x} = \text{RHS } \checkmark$$