37) $\dfrac{\sin(x+y)-\sin(x-y)}{\cos(x+y)+\cos(x-y)} = \tan y.$

LHS: $\dfrac{\cancel{\sin x\cos y} + \sin y\cos x \;\cancel{-\sin x\cos y} + \sin y\cos x}{\cos x\cos y \;\cancel{-\sin x\sin y} + \cos x\cos y \;\cancel{+\sin x\sin y}}$

$= \dfrac{2\sin y\cos x}{2\cos x\cos y} = \tan y = \text{RHS}$ ✓

35) $\tan x - \tan y = \dfrac{\sin(x-y)}{\cos x \cos y}$

RHS: $\dfrac{\sin x \cos y - \sin y \cos x}{\cos x \cos y}$

$= \dfrac{\sin x \cos y}{\cos x \cos y} - \dfrac{\sin y \cos x}{\cos x \cos y}$

$= \tan x - \tan y = \text{LHS}$ ✓

19) $\tan\left(\frac{\pi}{2}-u\right)=\cot u$

LHS: $\dfrac{\tan\frac{\pi}{2}-\tan u}{1+\tan\frac{\pi}{2}\tan u}$

$=\dfrac{\frac{\cot u}{\cot\frac{\pi}{2}}-\frac{1}{\cot u\cot\frac{\pi}{2}}}{\frac{\cot\frac{\pi}{2}\cot u}{\cot\frac{\pi}{2}\cot u}+\frac{1}{\cot\frac{\pi}{2}\cot u}}=\dfrac{\frac{\cot u}{\cot u\cot\frac{\pi}{2}}}{\frac{1}{\cot\frac{\pi}{2}\cot u}}$

$=\dfrac{\cot u}{\cot u\cot\frac{\pi}{2}}\cdot\dfrac{\cot\frac{\pi}{2}\cot u}{1}$

$=\cot u$

$=$ RHS ✓

34) $\cot(x+y) = \dfrac{\cot x \cot y - 1}{\cot x + \cot y}$

LHS $= \dfrac{1}{\tan(x+y)} = \dfrac{1}{\frac{\tan x + \tan y}{1 - \tan x \tan y}} = \dfrac{1 - \tan x \tan y}{\tan x + \tan y}$

$\cot x \cot y$

$= \dfrac{\frac{1}{\cot x} \quad \frac{1}{\cot y}}{\frac{\cot y}{\cot y \cot x} + \frac{1}{\cot y}}$

$= \dfrac{\frac{\cot x \cot y - 1}{\cot x \cot y}}{\frac{\cot y + \cot x}{\cot x \cot y}}$

7) $\tan(x - \pi) = \tan x$

LHS: $\dfrac{\tan x - \cancel{\tan \pi}^{0}}{1 + \cancel{\tan x \tan \pi}^{0}} = \dfrac{\tan x}{1} = \tan x = RHS$ ✓

37) $$\frac{\sin(x+y)-\sin(x-y)}{\cos(x+y)+\cos(x-y)} = \tan y$$

LHS: $$\frac{\sin x\cos y + \sin y\cos x - (\sin x\cos y - \sin y\cos x)}{\cos x\cos y - \sin x\sin y + \cos x\cos y + \sin x\sin y}$$

$$= \frac{2\sin y\cos x}{2\cos x\cos y} = \tan y = \text{RHS} \checkmark$$

33) $\cot(x-y) = \dfrac{\cot x \cot y + 1}{\cot y - \cot x}$

LHS: $\dfrac{1}{\tan(x-y)} = \dfrac{1}{\frac{\tan x - \tan y}{1+\tan x \tan y}} = \dfrac{1+\tan x \tan y}{\tan x - \tan y} = \dfrac{1+\frac{1}{\cot x}\frac{1}{\cot y}}{\frac{1}{\cot x} - \frac{1}{\cot y}}$

$= \dfrac{\frac{\cot x \cot y + 1}{\cot x \cot y}}{\frac{\cot y - \cot x}{\cot x \cot y}} = \dfrac{\cot x \cot y + 1}{\cot y - \cot x} \cdot \dfrac{\cot x \cot y}{\cot y - \cot x} = \dfrac{\cot x \cot y + 1}{\cot y - \cot x}$

$=$ RHS ✓