$(x-3)(x+3)$

Warm Up 2/26

Verify the identity.

$$\frac{\sec x}{\sec x - \tan x} = \sec x\,(\sec x + \tan x)$$

LHS = $\dfrac{\sec x}{\sec x - \tan x}$

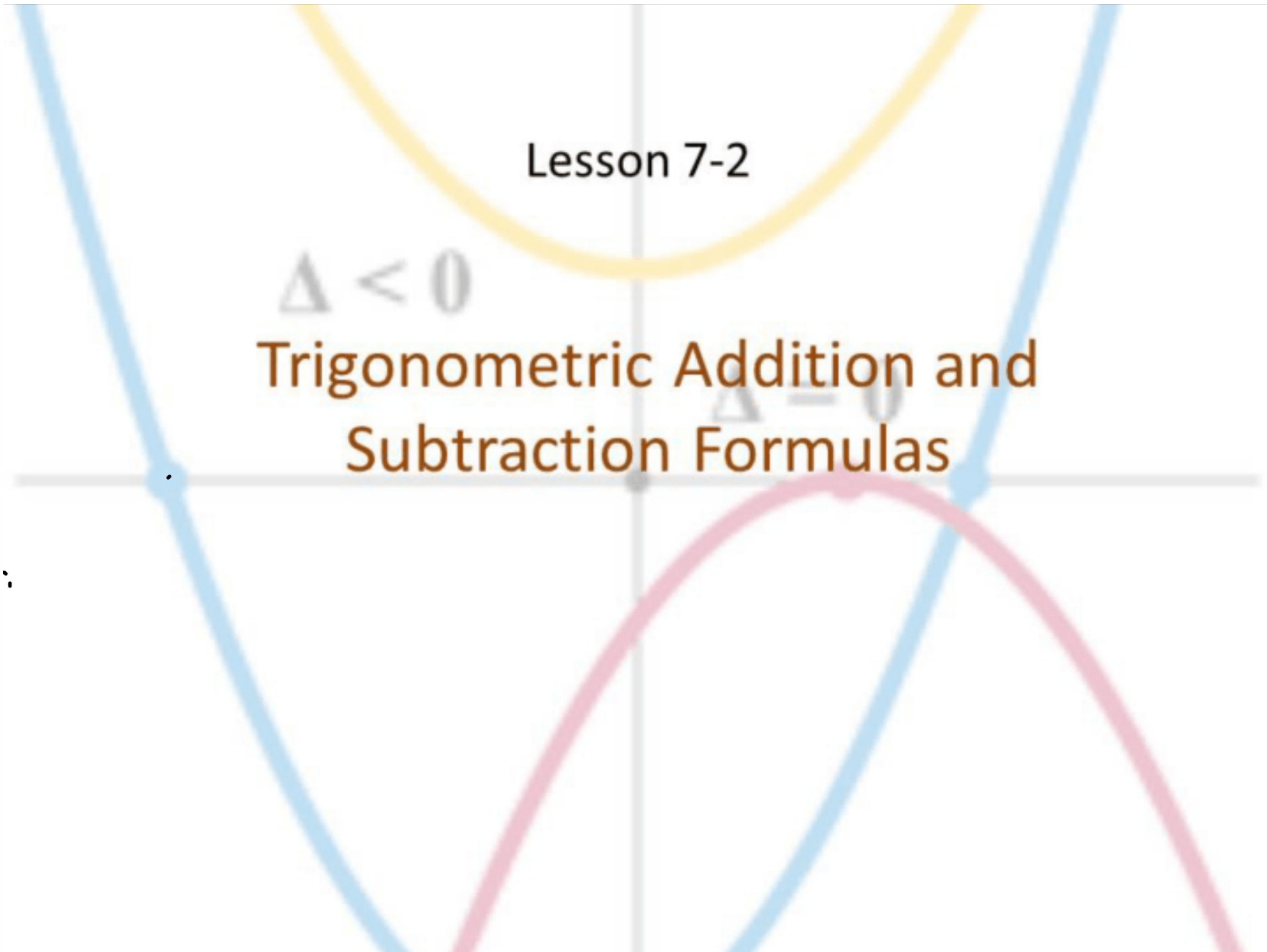
$= \dfrac{\frac{1}{\cos x}}{\frac{1}{\cos x} - \frac{\sin x}{\cos x}} = \dfrac{\frac{1}{\cos x}}{\frac{1-\sin x}{\cos x}}$

$= \dfrac{1}{\cancel{\cos x}} \cdot \dfrac{\cancel{\cos x}}{1-\sin x} = \dfrac{1}{1-\sin x} \cdot \dfrac{(1+\sin x)}{(1+\sin x)} = \dfrac{1+\sin x}{\cos^2 x}$

$= \dfrac{1+\sin x}{1-\sin^2 x} = \dfrac{1+\sin x}{\cos^2 x}$

RHS = $\sec x(\sec x + \tan x) = \sec^2 x + \sec x \tan x$

$= \dfrac{1}{\cos^2 x} + \dfrac{1}{\cos x} \cdot \dfrac{\sin x}{\cos x} = \dfrac{1}{\cos^2 x} + \dfrac{\sin x}{\cos^2 x}.$


Lesson 7-2
Δ < 0
Trigonometric Addition and
Subtraction Formulas

Objective

Students will…

- Be able to know the addition and subtraction formulas for sine, cosine, and tangent.
- Be able to use addition and subtraction formulas to evaluate trig functions and to prove or verify identities.

Trigonometric Identities

Try to fill these in from memory as much as possible!

$\csc x = \frac{1}{\sin x}$ $\sec x = \frac{1}{\cos x}$ $\cot x = \frac{1}{\tan x}$

$\tan x = \frac{\sin x}{\cos x}$ $\cot x = \frac{\cos x}{\sin x}$

Pythagorean Identity: $\sin^2 x + \cos^2 x = 1$

From this, we also get:

$\sin^2 x = 1 - \cos^2 x$ and $\cos^2 x = 1 - \sin^2 x$

$\tan^2 x + 1 = \sec^2 x$ and $1 + \cot^2 x = \csc^2 x$

$\tan^2 x = \sec^2 x - 1$ and $\cot^2 x = \csc^2 x - 1$

$\sec^2 x - \tan^2 x = 1$ and $\csc^2 x - \cot^2 x = 1$

Addition and Subtraction Formulas

Formulas for Sine:

$$\sin(s+t) = \sin s \cos t + \cos s \sin t$$
$$\sin(s-t) = \sin s \cos t - \cos s \sin t$$

Formulas for Cosine:

$$\cos(s+t) = \cos s \cos t - \sin s \sin t$$
$$\cos(s-t) = \cos s \cos t + \sin s \sin t$$

Formulas for Tangent:

$$\tan(s+t) = \frac{\tan s + \tan t}{1 - \tan s \tan t}$$

$$\tan(s-t) = \frac{\tan s - \tan t}{1 + \tan s \tan t}$$

$(\cos, \sin)$

Using Addition and Subtraction Formulas

$x \cdot y = xy$
$x + y = x + y.$

Find the **exact** value of each expression.

$\pi/4$ $\pi/3$ $30°, 45°, 60°, 90°$

$0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}$ $\frac{3}{12}$ $\frac{4}{12}$

a) $\cos 75°$ b) $\cos\frac{\pi}{12}$ c) $\cos 15°$

$\cos(60-45)$

a)

$\cos(45 +$

$= \cos(45)\cos($

$= \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$

$= \frac{\sqrt{6}}{4}$

$= \boxed{\frac{\sqrt{6}-\sqrt{2}}{4}}$

b)

$\cos\left(\frac{\pi}{3} - \frac{\pi}{4}\right)$

$= \cos\left(\frac{\pi}{3}\right)\cos\left(\frac{\pi}{4}\right)$

$= \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$

$= \frac{\sqrt{2}}{4} +$

c)

$\cos(45-30)$

$= \cos(45)\cos(30) + \sin(45)\sin(30)$

$= \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)$

$= \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} = \boxed{\frac{\sqrt{6}+\sqrt{2}}{4}}$

Example

Find the exact value of the expression:

$\sin \overset{s}{20^\circ} \cos \overset{t}{40^\circ} + \cos \overset{s}{20^\circ} \sin \overset{t}{40^\circ}$

$= \sin(20^\circ + 40^\circ)$

$= \sin(60^\circ)$

$= \boxed{\frac{\sqrt{3}}{2}}$

Example

Find the exact value of the expression:

$\sin 10° \cos 50° + \cos 10° \sin 50°$

$\sin(10+50)$

$= \sin(60)$

$= \boxed{\frac{\sqrt{3}}{2}}$

$\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

Homework Problems

$\frac{\sqrt{2}}{2} / \frac{\sqrt{2}}{2}$, $\frac{3}{12}$, $\frac{4}{12}$

$\frac{\sin}{\cos}$ $\frac{\sqrt{3}/2}{1/2} = \frac{\sqrt{3}}{2} \cdot \frac{2}{1} = \sqrt{3}$ $\frac{\pi}{2}, \frac{\pi}{4}, \frac{\pi}{6}, \frac{\pi}{3}, \ldots$

Find the **exact** value of each expression.

$30°, 60°, 45°, 90°$

3. $\cos 105°$

$\cos(60° + 45°)$

$= \cos(60°)\cos(45°) - \sin(60°)$

$= \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$

9. $\tan\left(-\frac{\pi}{12}\right) = -\tan\left(\frac{\pi}{12}\right)$

$= -\tan\left(\frac{\pi}{3} - \frac{\pi}{4}\right)$

$= -\dfrac{\tan\left(\frac{\pi}{3}\right) - \tan\left(\frac{\pi}{4}\right)}{1 + \tan\left(\frac{\pi}{3}\right)\tan\left(\frac{\pi}{4}\right)}$

$= -\dfrac{\sqrt{3} - 1}{1 + (\sqrt{3})(1)} = \boxed{-\dfrac{\sqrt{3} - 1}{1 + \sqrt{3}}}$

Homework Problems

Find the **exact** value of each expression.

15. $\cos\overset{s}{\frac{3\pi}{7}}\cos\overset{t}{\frac{2\pi}{21}}+\sin\frac{3\pi}{7}\sin\frac{2\pi}{21}$

$$=\cos\left(\frac{3\cdot 3\pi}{3\cdot 7}-\frac{2\pi}{21}\right)=\cos\left(\frac{9\pi}{21}-\frac{2\pi}{21}\right)=\cos\left(\frac{7\pi}{21}\right)$$

$$=\cos\left(\frac{\pi}{3}\right)=\boxed{\frac{1}{2}}$$

Homework Problems

Find the **exact** value of each expression.

16. $$\frac{\tan\overset{s}{\frac{\pi}{18}}+\tan\overset{t}{\frac{\pi}{9}}}{1-\tan\frac{\pi}{18}\tan\frac{\pi}{9}} = \tan\left(\frac{\pi}{18}+\frac{\pi\cdot 2}{9\cdot 2}\right) = \tan\left(\frac{\pi}{18}+\frac{2\pi}{18}\right)$$

$\frac{y}{x}$

$$= \tan\left(\frac{3\pi}{18}\right) = \tan\left(\frac{\pi}{6}\right) = \frac{1/2}{\frac{\sqrt{3}}{2}} = \frac{1}{\cancel{2}}\cdot\frac{\cancel{2}}{\sqrt{3}}$$

$$= \frac{1\cdot\sqrt{3}}{\sqrt{3}\cdot\sqrt{3}}$$

$$= \boxed{\frac{\sqrt{3}}{3}}$$

Homework 2/26

TB pg. 539 #1-17 (odd)

$$\tan = \frac{\sin}{\cos} = \frac{y}{x}$$

$$\cot = \frac{\cos}{\sin} = \frac{x}{y} \quad \frac{3}{12} + \frac{16}{12}$$

$$\sec = \frac{1}{\cos} = \frac{1}{x} \quad \frac{3}{12} \quad \left(\frac{4}{12}\right) \quad \frac{\pi}{3} \quad \frac{\pi}{4}$$

$$\frac{19\pi}{12}$$

$$= \sin\left(\frac{\pi}{4} + \frac{4\pi}{3}\right)$$