Lesson 7-3

Double-Angle, and Half-Angle Formulas II

Objective

Students will…

- Be able to apply the Double-Angle and Half-Angle formulas to verify identities.

Double-Angle Formulas

The following formulas are direct results of addition and subtraction formulas. **Double-Angle** formulas allows us to find the values of the trigonometric functions at $2x$ from their values at x.

Double-Angle Formulas:

For Sine: $\sin(2x) = 2\sin x \cos x$

For Cosine: $\cos(2x) = \cos^2 x - \sin^2 x$

$$= 1 - 2\sin^2 x$$

$$= 2\cos^2 x - 1$$

For Tangent: $\tan(2x) = \frac{2\tan x}{1-\tan^2 x}$

Proof of Double-Angle Formulas

Prove the formula: $\cos(2x) = \cos^2 x - \sin^2 x$

$\cos(x+x) = \cos x \cos x - \sin x \sin x$

$= \cos^2 x - \sin^2 x.$

Show that $\cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$

$= \cos^2 x - \sin^2 x = \cos^2 x - (1 - \cos^2 x) = 2\cos^2 x - 1$

$= \cos^2 x - \sin^2 x = (1 - \sin^2 x) - \sin^2 x = 1 - 2\sin^2 x.$

Proof of Double-Angle Formulas

Prove the formula: $\sin(2x) = 2\sin x \cos x$

$\sin(x+x) = \sin x \cos x + \cos x \sin x$

$= 2 \sin x \cos x$

Prove the formula: $\tan(2x) = \frac{2\tan x}{1-\tan^2 x}$

$\tan(x+x) = \frac{\tan x + \tan x}{1 - \tan x \tan x} = \frac{2\tan x}{1-\tan^2 x}.$

Half-Angle Formulas

The next set of formulas relate the values of trig functions at $\frac{1}{2}x$ to their values at x. They are known as the **Half-Angle Formulas**.

Half-Angle Formulas:

$$\sin\frac{u}{2} = \pm\sqrt{\frac{1-\cos u}{2}} \qquad \cos\frac{u}{2} = \pm\sqrt{\frac{1+\cos u}{2}}$$

$$\tan\frac{u}{2} = \frac{1-\cos u}{\sin u} = \frac{\sin u}{1+\cos u}$$

*The choice of $+$ or $-$ depends on which quadrant $\frac{u}{2}$ lies in.

Guidelines for Proving Identities

1. Always look for opportunities to use the Sum-to-Product formulas, before applying the Double-Angle formulas.

2. When either can be used, it's **usually** best to use the Double-Angle formulas, rather than the Half-Angle formulas. Half-Angle formulas are rarely used when proving identities.

3. With regards to the Double-Angle formulas, always look for multiples of 2. When whole numbers are doubled, they are always even (i.e. multiples of 2!).

ex: $\sin 8x = \sin 2(4x)$

Example

Prove the identity: $\frac{\sin 3x}{\sin x \cos x} = 4\cos x - \sec x$

LHS: $\frac{\sin(2x+x)}{\sin x \cos x} = \frac{\sin 2x \cos x + \cos 2x \sin x}{\sin x \cos x}$

$= \frac{\sin 2x \cos x}{\sin x \cos x} + \frac{\cos 2x \sin x}{\sin x \cos x} = \frac{2 \sin x \cos x}{\sin x} + \frac{2\cos^2 x - 1}{\cos x}$

$= 2\cos x + \frac{2\cos^2 x}{\cos x} - \frac{1}{\cos x}$

$= 2\cos x + 2\cos x - \sec x = 4\cos x - \sec x$

$=$ RHS ✓

Homework Problems

Prove the identity.

60. $\sin 8x = 2 \sin 4x \cos 4x$

Homework Problems

Prove the identity.

62. $\frac{2\tan x}{1+\tan^2 x} = \sin 2x$

Homework Problems

Prove the identity.

64. $\frac{1+\sin 2x}{\sin 2x} = 1 + \frac{1}{2}\sec x \csc x$

Homework Problems

66. $\cot 2x = \frac{1-\tan^2 x}{2 \tan x}$

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