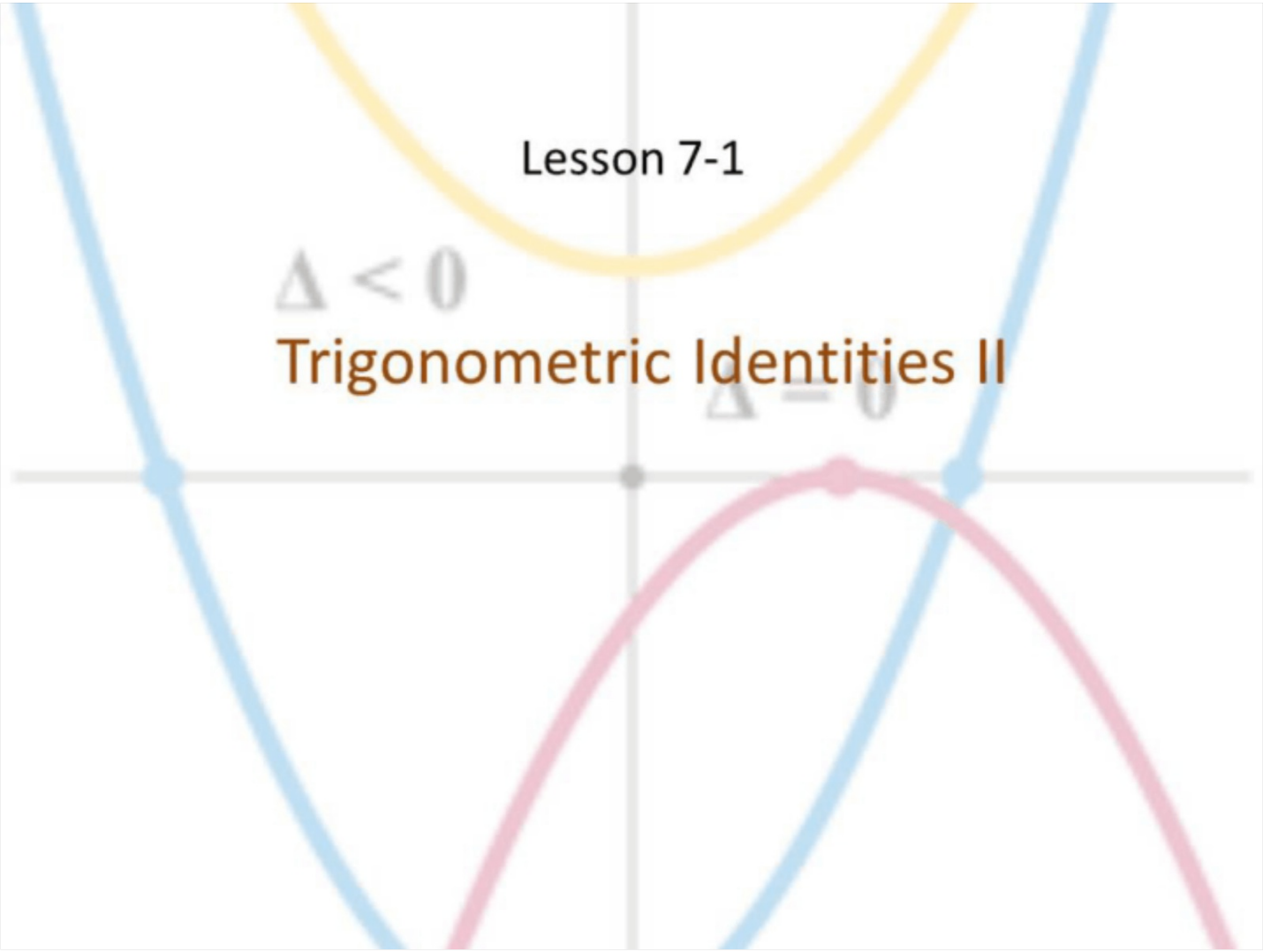
Warm Up 2/11

Simplify the trigonometric expression.

1. $\frac{\sec x - \cos x}{\tan x}$

$$= \frac{\frac{1}{\cos x} - \frac{\cos x \cos x}{1 \cos x}}{\frac{\sin x}{\cos x}} = \frac{\frac{1-\cos^2 x}{\cos x}}{\frac{\sin x}{\cos x}}$$

$$= \frac{\sin^2 x}{\cos x} \cdot \frac{\cos x}{\sin x} = \boxed{\sin x}$$


Lesson 7-1
Δ < 0
Trigonometric Identities II
Δ = 0

Objective

Students will…

- Be able to prove or verify trigonometric identities.
- Be able to simplify expressions using trigonometric substitution.

Trigonometric Identities

Before we get any deep into trig analysis, we must first recall some of the basic trigonometric identities and definitions. Primarily,

$$\csc x = \frac{1}{\sin x} \qquad \sec x = \frac{1}{\cos x} \qquad \cot x = \frac{1}{\tan x}$$

$$\tan x = \frac{\sin x}{\cos x} \qquad \cot x = \frac{\cos x}{\sin x}$$

Pythagorean Identity: $\sin^2 x + \cos^2 x = 1$

From this, we also get:

$\sin^2 x = 1 - \cos^2 x$ and $\cos^2 x = 1 - \sin^2 x$

$\tan^2 x + 1 = \sec^2 x$ and $1 + \cot^2 x = \csc^2 x$

$\tan^2 x = \sec^2 x - 1$ and $\cot^2 x = \csc^2 x - 1$

Methods for Proving Identities

One of the main components of trig analysis is to prove identities. There are two different methods for proving identities:

$2x-3=x+4$ (+3) (+3)

I. Rewrite **one** of the sides to match the other side.

Ex.

$$x+3=6\left(\frac{1}{6}x+\frac{11}{6}-\frac{13}{6}+\frac{5}{6}\right)$$

$=6(1/6x+3/6)$

$=x+3$

II. Modify **both** sides until they are the same.

Ex.

$$3(2x-1)=2x+2\left(2x-\frac{3}{2}\right)$$

$=6x-3$ $=2x+4x-3$

$=6x-3$

Guidelines for Proving Identities

Furthermore, we have some guidelines/tips for proving identities.

1. **Focus on the fractions**: More often than not, identity proofs are more easily done when you work with the side that involves a fraction.

2. **Pick the more "complicated" side**: It's easier to modify the sides that has less sines or cosines. Generally, rewriting everything as sine or cosine can help you when you are "stuck."

3. **Use the Known Identities!**: Use algebra and the identities are already known to you. Look to combine multiple fractions into one with a common denominator.

Example

Prove/Verify the identity: $\cos\theta\,(\sec\theta - \cos\theta) = \sin^2\theta$

LHS RHS

LHS: $\cos\theta\sec\theta - \cos^2\theta = \cos\theta\,\frac{1}{\cos\theta} - \cos^2\theta$

$= 1 - \cos^2\theta = \sin^2\theta = \text{RHS.}$ ✓

Example

Prove/Verify the identity: $\cos x \tan x = \sin x$

LHS: $\cancel{\cos x}\,\dfrac{\sin x}{\cancel{\cos x}} = \sin x = \text{RHS}.$

Homework Problems

Verify the identity:

31. $\sin B + \cos B \cot B = \csc B$

$$\text{HS} = \sin B + \frac{\cos B}{1}\cdot\frac{\cos B}{\sin B} = \frac{\sin B}{\sin B}\sin B + \frac{\cos^2 B}{\sin B} = \frac{\sin^2 B}{\sin B} + \frac{\cos^2 B}{\sin B}$$

$$= \frac{\sin^2 B + \cos^2 B}{\sin B} = \frac{1}{\sin B} = \csc B = \text{RHS}$$

✓

Homework Problems

Verify the identity :

29. $\frac{\tan y}{\csc y} = \sec y - \cos y$

LHS: $\frac{\frac{\sin y}{\cos y}}{\frac{1}{\sin y}} = \frac{\sin y}{\cos y} \cdot \frac{\sin y}{1} = \frac{\sin^2 y}{\cos y} = \frac{1-\cos^2 y}{\cos y}$

$= \frac{1}{\cos y} - \frac{\cos^2 y}{\cos y} = \sec y - \cos y = RHS$

Homework Problems

Verify the identity :

35. $\tan\theta + \cot\theta = \sec\theta \csc\theta$

Homework Problems

Verify the identity :

51. $\frac{1-\cos\alpha}{\sin\alpha} = \frac{\sin\alpha}{1+\cos\alpha}$

Homework Problems

Verify the identity :

85. $\frac{1+\sin x}{1-\sin x} = (\tan x + \sec x)^2$

Homework 2/11

TB pg. 533 #29, 31, 35, 37, 41, 49, 51, 59, 73, 79, 85