Warm Up 11/17

Solve.

1) $2x - 6 = 6$

$X = 6$

2) $2x + 5x - 3 = 9$

$X = \frac{12}{7}$

3) $4x^2 = 100$

$x = \pm 5$

4) $\frac{x}{6} = 8$

$X = 48$

5) $\frac{4}{x} = 2$

$X = 2$

6) $log_x 9 = 2$

$x = 3$

Lesson 4-4

Exponential and Logarithmic Equations

Objective

Students will…

- Be able to apply the inverse relationship between exponential and logarithmic functions and solve their equations algebraically.

Solving Exponential and Logarithmic Equations

So far in this chapter, we've learned various components and techniques involving exponential and logarithmic functions. Knowing about these characteristics and techniques can be useful when it comes to solving equations.

An **exponential equation** is one in which the variable occurs in the exponent. For example, $2^x = 7$.

A **logarithmic equation** is one in which a logarithm of the variable occurs. For example, $log_2(x + 2) = 5$

Although solving these equations may appear to be difficult, if we simply use their inverse relationship, they become quite easy.

Inverses

Recall that exponential and logarithmic functions are inverses of each other. Just the way we solve any algebraic equations, our goal is to solve for that particular variable (i.e. isolate the variable) by taking various inverses.

Ex. $2x - \cancel{6} = 6$ $x = 6$

$$\begin{array}{c} +6 \quad +6 \\ \hline 2x = 12 \\ \frac{2x}{2} = \frac{12}{2} \end{array}$$

To "undo" the subtraction, we added. To "undo" multiplication, we divided. This worked, of course, because they are inverses to each other.

We would need to do something similar. To "undo" the exponential, we need to use logarithms. Vice-versa, to "undo" logarithms, we take the exponential.

Examples

1. $3^{x+2} = 7$

$\cancel{\log_3}(\cancel{3}^{x+2}) = \log_3(7)$

$x + 2 = \log_3(7) - 2$

-2

$x = \log_3(7) - 2 \approx 0.229$

2. $3^{x+3} = 5$

$\cancel{\log_3}(\cancel{3}^{x+3}) = \log_3(5)$

$x + 3 = \log_3(5)$

$x = \log_3(5) - 3 \approx -1.535$

3. $\frac{\cancel{8}e^{2x}}{\cancel{8}} = \frac{20}{8}$

$e^{2x} = \frac{5}{2}$

$\cancel{\ln}(\cancel{e}^{2x}) = \ln\left(\frac{5}{2}\right)$

$\frac{\cancel{2}x}{\cancel{2}} = \frac{\ln\left(\frac{5}{2}\right)}{2} \approx 0.458$

3. $10^{56x} = 7$

$\cancel{\log}(\cancel{10}^{56x}) = \log(7)$

$\frac{56x}{56} = \frac{\log(7)}{56} \approx 0.0151$

Examples

5. $log_2(x+2) = 5$

$2^{\wedge}$ $\quad$ $2^{\wedge}$

$x+2=32$

$x=30$

6. $\ln x = 8$

$e^{\wedge}$ $\quad$ $e^{\wedge}$

$x = e^8 \approx 2980.96$

7. $log_2(25-x) = 3$

$2^{\wedge}$ $\quad$ $2^{\wedge}$

$25-x=8$

$x=17$

8. $log_2(22-x) = 3$

$2^{\wedge}$ $\quad$ $2^{\wedge}$

$22-x=8$

$x=14$

Examples

laws of log 9. $\log(x+2)+\log(x-1)=1$

$\log((x+2)(x-1))=1$

10^1 $(x+2)(x-1)=10$

$x^2+x-2=10$

$x^2+x-12=0$ -12, 1

$(x+4)(x-3)=0$

$x=\cancel{-4},\ 3$

10. $3xe^x+x^2e^x=0$

$(xe^x)(3+x)=0$

$(x)(e^x)=0$ $x=0$ $\cancel{e^x=0}$

or

$3+x=0$

$x=-3$

10. $e^{2x}-e^x-6=0$

-6; -3, 2; -1

$(e^x-3)(e^x+2)=0$

$e^x=3$ on $\cancel{e^x=-2}$

$\ln(e^x)=\ln(3)$

$\boxed{x\approx 1.099}$

In Closing

Expand or combine the following using the laws of logarithms and check your answers with a partner.

1) $log_4 \frac{x}{2}$

2) $\log 12 + \frac{1}{2}\log 7 - \log 2$

Homework 11/17

TB pg. 366-367 #1-13(e.o.o), 25, 31, 35, 39, 43, 49