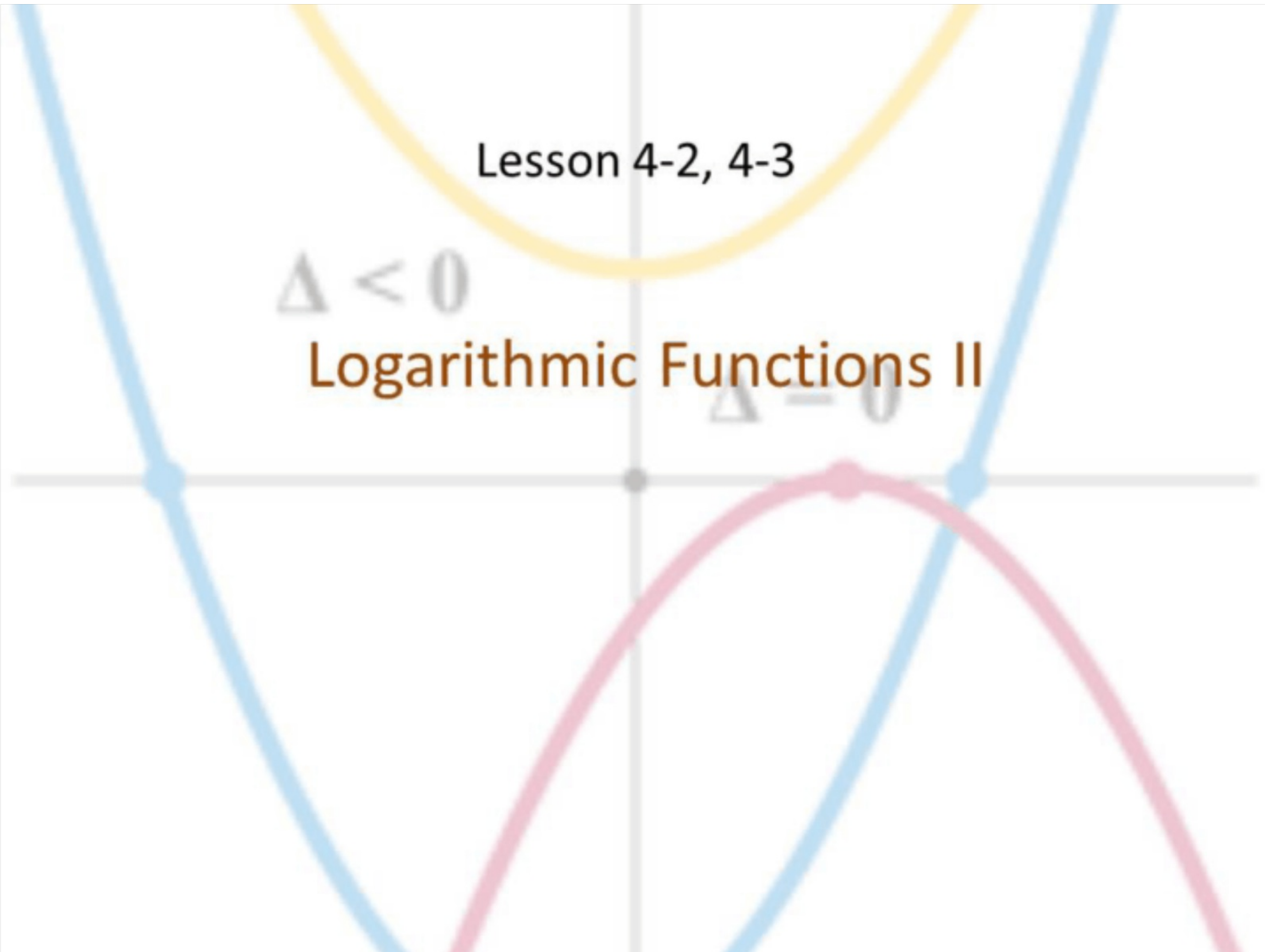
Lesson 4-2, 4-3
Logarithmic Functions II
$\Delta < 0$
$\Delta = 0$

Objective

Students will…

- Be able to define natural logarithmic function.
- Be able to know and apply the properties of natural logarithms.
- Be able to know and use the Laws of Logarithms.

Natural Logarithms

We've learned that any logarithm with base 10 is known as the *common* logarithm, without the base written. In our previous section of exponential function, we learned about a very special number denoted, e. Naturally (no pun intended as we'll see), logarithms with base e is also considered special, and it is given a special name.

Natural Logarithm- The logarithm with base e is called the **natural logarithm** and is denoted by **ln**:

$$\ln x = log_e x$$

The Inverse of Exponential Function

Like all other exponential and logarithmic functions, the natural logarithmic function $y = \ln x$ is the inverse function of the exponential function $y = e^x$. Hence, by definition we have

$$\ln x = y \leftrightarrow e^y = x$$

Example:

$$e^6 \approx 403.43 \quad \rightarrow \quad \ln 403.43 \approx 6$$

$$\ln 8 \approx 2.08 \quad \rightarrow \quad e^{2.08} \approx 8$$

Properties of Natural Logarithms

We have learned about some of the basic properties of logarithms. Always remember that, although it's given a special name, natural logarithms is still a logarithmic function! Thus, the properties of natural logarithm naturally (again, no pun intended ☺) follow the properties of logarithms. Simply replace a with e and log_a with ln.

Property	Reason
1. $\ln 1 = 0$	Anything raised to the zero power is 1
2. $\ln e = 1$	Anything raised to the 1st power is itself
3. $\ln e^x = x$	e raised to the x power is e^x
4. $e^{\ln x} = x$	$\ln x$ is the power to which e must be raised to get x

Examples

For base 5...

By property 1:

$$\ln 1 = 0$$

By property 2:

$$\ln e = 1$$

By property 3:

$$\ln e^8 = 8$$

By property 4:

$$e^{\ln 12} = 12$$

Change of Base

ex. $\log_3 9 = 2$

The last thing we need to cover in this section is the change of base formula. There were two "special" bases for logarithms: base 10 and e. Thus, for computing logarithms using a calculator, our goal is to **change the base** of our expression to either base 10 or e. This can be done by:

Change of Base Formula:

$$log_b x = \frac{log_a x}{log_a b}$$

ex. $\log_3 6 = \frac{\ln 6}{\ln 3} = \frac{\log 6}{\log 3}$

This formula allows us to change the base of our logarithm to any base we want to. However, as mentioned, we want to change the base to either base 10 or e. Be aware that base e is used more.

Examples

Use the change of base formula and calculator to evaluate.

a) $log_5 8$

$= \frac{\ln 8}{\ln 5} \approx 1.292$

b) $log_9 20$

$\frac{\log 20}{\log 9} \approx 1.3634$

Laws of Exponents

When we learned about exponents, we learned that there are certain set of rules regarding exponents, called **The Laws of Exponents**.

For example, when we multiply the same base number with exponents, we simply add the exponents together (i.e. $a^3a^4 = a^{3+4} = a^7$).

On the other and when we divide the same base number with exponents, we would subtract the exponents (i.e. $\frac{a^8}{a^7} = a^{8-7} = a^1$).

Lastly, when we take the exponent to an exponent, we would multiply the two exponents together (i.e. $(a^6)^7 = a^{6\times7} = a^{42}$)

Laws of Logarithms

Now, recall that the answers (the output) of logarithms are exponents. For example, we now know that $log_4 x = 6$ is equivalent to $4^6 = x$. Hence, there also exists **Laws of Logarithms**, much similar to the Laws of Exponents.

Laws of Logarithms- Let a be a positive number, with $a \neq 1$. Let A, B, and C be any real numbers with $A > 0$ and $B > 0$.

"Condensed" "Expanded".

Laws mult. Add.

1. $log_a(AB) = log_a A + log_a B$

Div. Subt.

2. $log_a\left(\frac{A}{B}\right) = log_a A - log_a B$
3. $log_a(A^C) = C log_a A$

Just like the Laws of Exponents, it is **imperative** to have the **same** base!

Examples

By Law 1,

$$log_2(8 * 32) = log_2(256) = 8 = (3 + 5) = log_2(8) + log_2(32)$$

By law 2,

$$log_2\left(\frac{32}{8}\right) = log_2(4) = 2 = (5 - 3) = log_2(32) - log_2(8)$$

By law 3,

$$log_2(4)^2 = log_2 16 = 4 = 2(2) = 2log_2(4)$$

Examples

Evaluate the following expressions.

a) $log_4 2 + log_4 32$

$= log_4(2 \cdot 32)$

$= log_4(64)$

$= \boxed{3}$

b) $log_6 4 + log_6 9$

$log_6(4 \cdot 9) = log_6(36)$

$= \boxed{2}$

c) $log_5 75 - log_5 3$

$log_5\left(\frac{75}{3}\right)$

$= log_5(25)$

$= \boxed{2}$

d) $log_2 80 - log_2 5$

$log_2\left(\frac{80}{5}\right) = log_2(16)$

$= \boxed{4}$

Examples

Use the Laws of Logarithms to expand each expression.

a) $log_2(6x)$

$= \log_2 6 + \log_2 x$

b) $log_5(x^3y^6)$

$= \log_5(x^3) + \log_5(y^6)$

$= 3\log_5(x) + 6\log_5(y)$

c) $\ln\left(\frac{ab}{\sqrt[3]{c}}\right)$

$= \ln(ab) - \ln(\sqrt[3]{c})$

$= \ln(a) + \ln(b) - \ln(c^{1/3})$

$= \ln(a) + \ln(b) - \frac{1}{3}\ln(c)$

Homework 11/14

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