Warm Up 12/02

Identify the base of each log or exponential function.

1) $2^x = 8$

2

2) $log_4 x = 2$

4

3) $3^x = 27$

3

4) $log_9 x = 625$

9

5) $e^x = 1$

e

6) $\log \mathrm{x} = 4$

10

Lesson 4-2

Logarithmic Functions II

Objective

Students will…

- Be able to define natural logarithmic function.
- Be able to know and apply the properties of natural logarithms.
- Be able to use calculators to compute natural logarithms.

Natural Logarithms

We've learned that any logarithm with base 10 is known as the *common* logarithm, without the base written. In our previous section of exponential function, we learned about a very special number denoted, e. Naturally (no pun intended as we'll see), logarithms with base e is also considered special, and it is given a special name.

Natural Logarithm- The logarithm with base e is called the **natural logarithm** and is denoted by **ln**:

$$\ln x = log_e x$$

The Inverse of Exponential Function

Like all other exponential and logarithmic functions, the natural logarithmic function $y = \ln x$ is the inverse function of the exponential function $y = e^x$. Hence, by definition we have

$$\ln x = y \leftrightarrow e^y = x$$

$\log_e x = y$

Example:

$$e^6 \approx 403.43 \quad \rightarrow \quad \ln 403.43 \approx 6$$

$$\ln 8 \approx 2.08 \quad \rightarrow \quad e^{2.08} \approx 8$$

Properties of Natural Logarithms

We have learned about some of the basic properties of logarithms. Always remember that, although it's given a special name, natural logarithms is still a logarithmic function! Thus, the properties of natural logarithm naturally (again, no pun intended ☺) follow the properties of logarithms. Simply replace a with e and log_a with ln.

Property	Reason
1. $\ln 1 = 0$	Anything raised to the zero power is 1
$\log_a a$	
2. $\cancel{\ln e} = 1$	Anything raised to the 1^{st} power is itself
$\log_e e$	
3. $\cancel{\ln} e^x = x$	e raised to the x power is e^x
4. $\cancel{e}^{\cancel{\ln} x} = x$	$\ln x$ is the power to which e must be raised to get x

Examples

For base 5…

By property 1:

$$\ln 1 = 0$$

By property 2:

$$\cancel{\ln e} = 1$$

By property 3:

$$\cancel{\ln e}^{8} = 8$$

By property 4:

$$\cancel{e}^{\ln 12} = 12$$

You try

By property 1:

$$\ln 1 = 0$$

By property 2:

$$\cancel{\ln e} = 1$$

By property 3:

$$\cancel{\ln e}^4 = 4$$

By property 4:

$$\cancel{e^{\ln}}\,^{19} = 19$$

Using a Calculator

For most logarithmic, as well as exponential functions, we've learned that having a calculator is a must. Computing natural logarithm on a calculator is easy. We simply need to find where the ln button is. Almost all calculators place e^x and ln together (usually "2nd" e^x).

Example:

To compute $\ln 5$, ~~we would input "2nd" e^x, then "5".~~

The answer should read: $\ln 5 = 1.60943791243 41$

In Closing

Compute the following natural logarithms using a calculator and check your answers with a partner.

1) $\ln 4 = 1.386$ 2) $2(\ln 9) = 4.394$ 3) $9(\ln 11) = 21.581$

Homework 12/02

TB pg. 349-350 #7, 8, 13, 14, 22a, 23b, 23c, 35, 36