Warm Up 12/5

Evaluate the following trig functions without using a calculator.

1. $\sin \frac{2\pi}{3}$

2. $\cos \frac{11\pi}{6}$

3. $\sec \frac{\pi}{4}$

4. $\tan \frac{4\pi}{3}$

Lesson 5-2c

Trigonometric Functions of Real Numbers III

Objective

Students will…

- Be able to rewrite trigonometric functions using the fundamental identities.
- Find all trigonometric functions from the value of one using the fundamental identities.

Soh Cah Toa

Recall that given a right triangle…

$$\sin t = \frac{opposite}{hypotenuse} \qquad \cos t = \frac{adjacent}{hypotenuse} \qquad \tan t = \frac{opposite}{adjacent}$$

$$\csc t = \frac{1}{\sin t} = \frac{hypotenuse}{opposite} \qquad \sec t = \frac{1}{\cos t} = \frac{hypotenuse}{adjacent}$$

$$\cot t = \frac{1}{\tan t} = \frac{adjacent}{opposite}$$

$$\tan t = \frac{\sin t}{\cos t} \qquad \frac{o}{\cancel{h}} \cdot \frac{\cancel{h}}{a}$$

We can use the properties of right triangles to figure out the rest of the trigonometric functions.

$\sin t = -\frac{4}{5}$ $\frac{o}{h}$ $\cos t = \frac{3}{5}$ $\frac{a}{h}$ $\tan t = -\frac{4}{3}$

$\csc t = -\frac{5}{4}$ $\sec t = \frac{5}{3}$

$\cot t = -\frac{3}{4}$

If $\cos t = -\frac{5}{13}$ $\frac{a}{h}$ and t is in quadrant II, find the values of all the trigonometric functions at t.

$$5^2 + b^2 = 13^2$$
$$b^2 = 144$$
$$b = 12$$

$$\sin t = \frac{12}{13}$$
$$\tan t = -\frac{12}{5}$$
$$\sec t = -\frac{13}{5}$$
$$\csc t = \frac{13}{12}$$
$$\cot t = -\frac{5}{12}$$

Trigonometric Functions

Recall from the definitions of trigonometric functions that…

$$\tan t = \frac{\sin t}{\cos t}$$

$$\cot t = \frac{1}{\tan t} = \frac{\cos t}{\sin t}$$

$$\sec t = \frac{1}{\cos t}$$

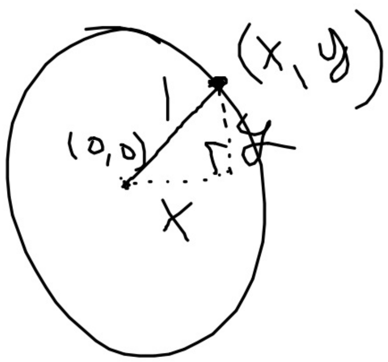
$$\csc t = \frac{1}{\sin t}$$

Coordinates on a Unit Circle

Now, also recall that on the unit circle, we defined the following:

$$\cos t = x \qquad \sin t = y \qquad \rightarrow \qquad (x, y) = (\cos t\,, \sin t)$$

Now, let's see how this can be applied on a unit circle.



$x^2 + y^2 = 1^2$

$x^2 + y^2 = 1$

$(\cos t)^2 + (\sin t)^2 = 1 \Rightarrow \boxed{\cos^2 t + \sin^2 t = 1}$

Pythagorean Identity

Pythagorean Identities

Hence, we can now conclude the following identities:

Pythagorean Identities: (Note: $sin^2t = (\sin t)^2$)

$sin^2t + cos^2t = 1$ $\qquad$ $tan^2t + 1 = sec^2t$ $\qquad$ $1 + cot^2t = csc^2t$

Also, moving some of these around using algebra:

$\sin t = \pm\sqrt{1 - cos^2t}$ $\qquad$ $\cos t = \pm\sqrt{1 - sin^2t}$

$\sin^2 t = 1 - \cos^2 t$ $\qquad$ $\cos^2 t = 1 - \sin^2 t$

Rewriting Trigonometric Functions

We can also rewrite trigonometric functions using others.

Example: Write $\tan t$ in terms of $\cos t$, where t is in quadrant III.

$$\tan t = \frac{\sin t}{\cos t} = \frac{\pm\sqrt{1-\cos^2 t}}{\cos t}$$

Examples

Write $\tan t$ in terms of $\sin t$, where t is in quadrant I.

$$\tan t = \frac{\sin t}{\cos t} = \frac{\sin t}{+\sqrt{1-\sin^2 t}}$$

Write $\sec t$ in terms of $\tan t$, where t is in quadrant II

$$\sqrt{\sec^2 t} = \sqrt{1+\tan^2 t}$$

$$\sec t = -\sqrt{1+\tan^2 t}$$

Homework 12/5

TB pg. 417 #53-61 (odd), 63, 64

Application of Pythagorean Identities

Although these identities will be used much more extensively in the future, we can still make use of them here.

Example: If $\cos t = \frac{3}{5}$ and t is in quadrant IV, find the values of all other trigonometric functions at t.

We need to first find $\sin t$. We use our identity: $\sin t = \pm\sqrt{1 - cos^2 t}$

$$\sin t = \pm\sqrt{1 - cos^2 t} = \pm\sqrt{1 - \left(\frac{3}{5}\right)^2} = \pm\sqrt{1 - \left(\frac{9}{25}\right)} = \pm\sqrt{\frac{16}{25}} = \pm\frac{4}{5}$$

However, since we are told that t is quadrant IV, $\sin t = -\frac{4}{5}$