Warm Up 12/2

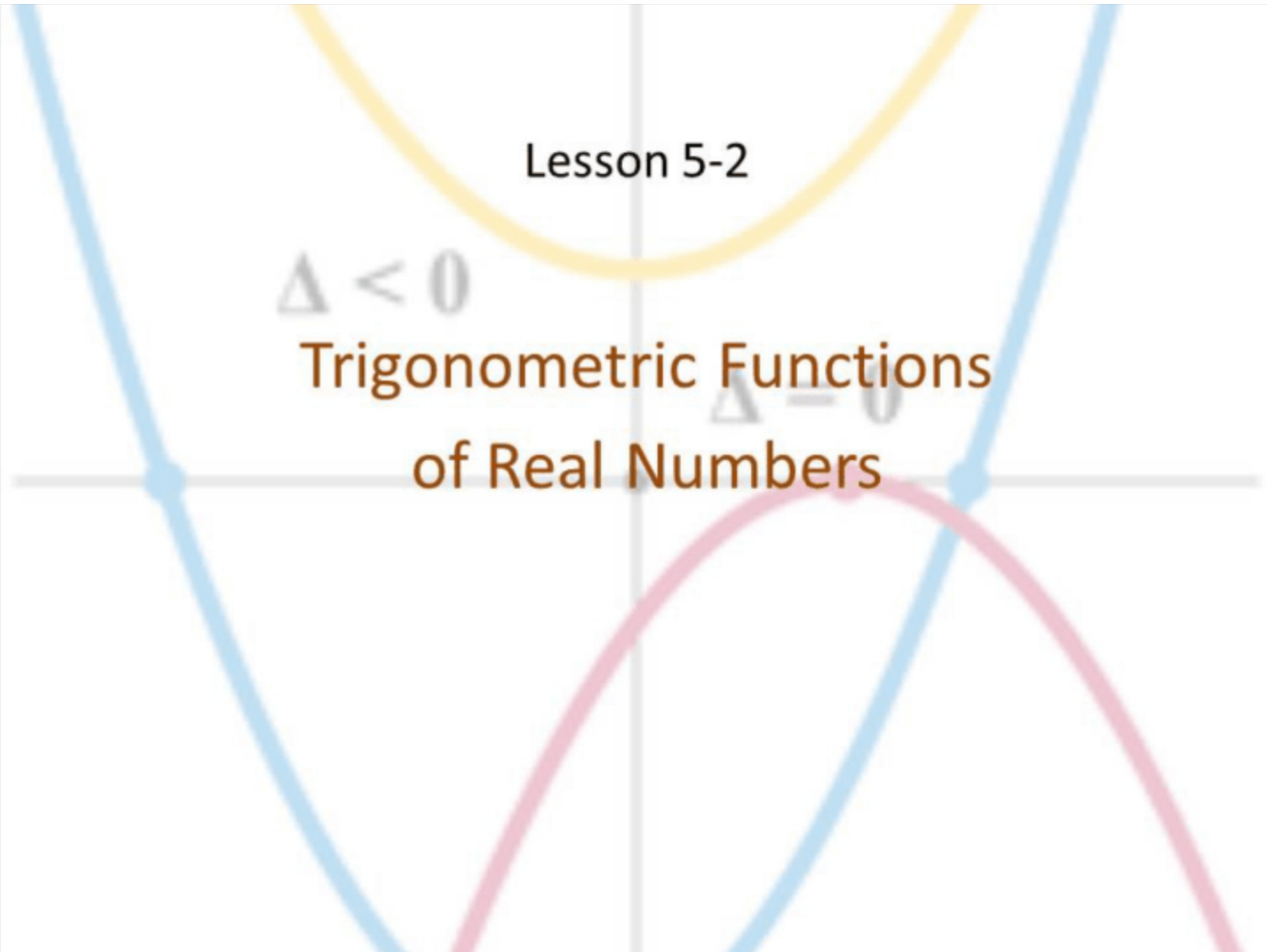
Write each fraction as a mixed number.

1) $\frac{4}{3} = 1\,\frac{1}{3}$ 2) $\frac{5}{4} = 1\,\frac{1}{4}$ 3) $\frac{7}{6} = 1\,\frac{1}{6}$

4) $\frac{3}{2} = 1\,\frac{1}{2}$ 5) $\frac{5}{3} = 1\,\frac{2}{3}$ 6) $\frac{11}{6} = 1\,\frac{5}{6}$

Are the following fractions greater or less than $\frac{1}{2}$?

7) $\frac{2}{3}$ $>$ 8) $\frac{1}{3}$ $<$ 9) $\frac{3}{4}$ $>$


Lesson 5-2
$\Delta < 0$
Trigonometric Functions
of Real Numbers

Objective

Students will…

- Be able to know that the coordinates of radians, $(x, y) = (\cos t\,, \sin t)$
- Be able to evaluate trigonometric functions in radians.

Trigonometric Functions

The concept of trigonometric functions can be defined in terms of the unit circle. The **definition of trigonometric functions** is as follows:

$\cos t = x$ - Coord.

$\sin t = y$ - Coord.

$\tan t = \dfrac{\sin t}{\cos t} = \dfrac{y}{x} \quad (x \neq 0)$

Secant

$\sec t = \dfrac{1}{\cos t} = \dfrac{1}{x}$

$(x \neq 0)$

Cosecant

$\csc t = \dfrac{1}{\sin t} = \dfrac{1}{y}$

$(y \neq 0)$

Cotangent

$\cot t = \dfrac{1}{\tan t} = \dfrac{x}{y} = \dfrac{\cos t}{\sin t}$

$(y \neq 0)$

Evaluating Trigonometric Functions

We have computed the (x, y) coordinate for each of the values on the unit circle. Based on our definition above, $(x, y) = (\cos t, \sin t)$. Consider the following units on the unit circle (Note that we are in **radians**).

$$0 = \overset{x,\,y}{(1, 0)} \quad \rightarrow \quad \overset{x}{\cos} 0 = 1 \quad , \ \overset{y}{\sin} 0 = 0 \quad , \ \tan 0 = \frac{\sin 0}{\cos 0} = \frac{y}{x} = \frac{0}{1} = \boxed{0}$$

$$\frac{\pi}{4} = \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right) \quad \rightarrow \quad \cos\frac{\pi}{4} = \frac{\sqrt{2}}{2} \quad , \ \sin\frac{\pi}{4} = \frac{\sqrt{2}}{2} \quad , \ \tan\frac{\pi}{4} = \frac{y}{x} = \boxed{1},$$

$$= \frac{\sqrt{2}}{2} \cdot \frac{2}{\sqrt{2}}$$

$\frac{\pi}{3} = (\frac{1}{2}, \frac{\sqrt{3}}{2}) \rightarrow$

$\sec \pi/3 = \boxed{2}$

$\csc \pi/3 = \frac{2 \cdot \sqrt{3}}{\sqrt{3} \cdot \sqrt{3}} = \boxed{\frac{2\sqrt{3}}{3}}$

$\cot \pi/3 = \frac{x}{y} = \frac{1}{\cancel{2}} \cdot \frac{\cancel{2}}{\sqrt{3}} = \frac{1}{\sqrt{3}} = \boxed{\frac{\sqrt{3}}{3}}$

$\frac{\pi}{4} = (\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}) \rightarrow$

$\sec \pi/4 = \frac{2}{\sqrt{2}} = \boxed{\sqrt{2}}$

$= \frac{\cancel{2}\sqrt{2}}{\cancel{2}}$

$\cot \pi/4 = \cancel{\frac{\sqrt{2}}{2}} \cdot \cancel{\frac{2}{\sqrt{2}}} = \boxed{1}$

$\csc \pi/4 = \boxed{\sqrt{2}}$

$\frac{\pi}{2} = (0, 1) \rightarrow$

$\sec \frac{\pi}{2} = \frac{1}{0} = \text{und.}$

ONE

$\csc \frac{\pi}{2} = 1$

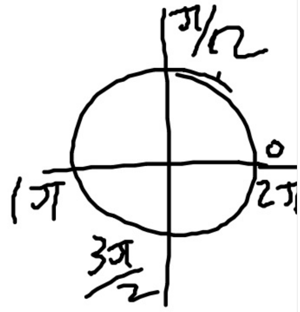
$\cot \frac{\pi}{2} = \frac{0}{1} = \boxed{0}$

$\frac{2\pi}{3} = \left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right) \rightarrow$

$\frac{5\pi}{4} = \left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right) \rightarrow$

$\frac{3\pi}{2} = (0, -1) \rightarrow$

$\frac{11\pi}{6} = \left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right) \rightarrow$



csc, sec, cot

For the following, give the values for $\csc t$, $\sec t$, and $\cot t$

$\frac{1}{3} = \frac{4\pi}{3} = \left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$

$\csc\frac{4\pi}{3} = -\frac{2}{\sqrt{3}} = \boxed{-\frac{2\sqrt{3}}{3}}$

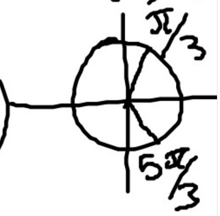
$\sec\frac{4\pi}{3} = \boxed{-2}$

$\cot\frac{4\pi}{3} = \frac{x}{y} = -\frac{1}{2} \cdot -\frac{2}{\sqrt{3}} = \frac{1}{\sqrt{3}} = \boxed{\frac{\sqrt{3}}{3}}$

Even-Odd Properties

Even: $f(-x) = f(x)$
Odd: $f(-x) = -f(x)$

Consider the following.



Odd $\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$ $\qquad$ $\sin\left(-\frac{\pi}{3}\right) = \sin\left(\frac{5\pi}{3}\right) = -\frac{\sqrt{3}}{2} = -\left(\sin\frac{\pi}{3}\right)$

Now, what about…

$\cos\frac{\pi}{3} = \frac{1}{2}$ $\qquad$ $\cos\left(-\frac{\pi}{3}\right) = \cos\left(\frac{5\pi}{3}\right) = \frac{1}{2} = \cos\frac{\pi}{3}$

Turns out, these results can be generalized.

Even-Odd Properties:

E $\cos(-t) = \cos t$ $\qquad$ O $\sin(-t) = -\sin t$ $\qquad$ O $\tan(-t) = -\tan t$

O $\csc(-t) = -\csc(t)$ $\qquad$ E $\sec(-t) = \sec t$ $\qquad$ O $\cot(-t) = -\cot t$

Examples

Use the Even-Odd Properties to evaluate the following.

O $\sin\left(-\frac{\pi}{6}\right) = -\sin \frac{\pi}{6} = -\frac{1}{2}$

E $\cos\left(-\frac{\pi}{4}\right) = \cos \frac{\pi}{4} = \frac{\sqrt{2}}{2}$

O $\cot\left(-\frac{5\pi}{6}\right) = -\cot \frac{5\pi}{6} = \neq \frac{\sqrt{3}}{\cancel{2}} \cdot \frac{\cancel{2}}{1} = \sqrt{3}$

O $\csc\left(-\frac{2\pi}{3}\right) = -\csc\left(\frac{2\pi}{3}\right) = -\left(\frac{2}{\sqrt{3}}\right) = \boxed{-\frac{2\sqrt{3}}{3}}$

$\tan\left(-\frac{11\pi}{6}\right) =$

$\sec\left(-\frac{\pi}{2}\right) =$

Homework 12/2

TB pg. 416 #3, 4, 6, 8, 9, 10, 14, 17, 18