Lesson 4-1a

Antiderivatives

And

Indefinite Integration

Objective

Students will…

- Be able to find the antiderivatives.
- Be able to use integral notation.
- Be able to come up with general and particular solutions to differential equations.

Antiderivatives

One of the key components in mathematics is being able to revert a process. So, naturally, if we can take the derivatives of a function, we should be able to "undo" it. This is what **antiderivatives** are.

Antiderivative- A function F is an antiderivative of f on an interval I if $F'(x) = f(x)$ for all x in I.

$F'(x) = 3x^2 = f(x)$

Ex. If $f(x) = 3x^2$, then possibly, $F(x) = x^3$

Note: Notice the word "possibly," because there are almost always multiple antiderivatives. From the above example,

If $f(x) = 3x^2$, then it is also possible that $F(x) = x^3 - 89$

$\frac{d}{dx} a^x = x a^{x-1}$

The "Anti-Power Rule"

$5 \cdot \frac{3}{5} = 3$

The Power Rule is probably the easiest and the simplest derivative rule. The antiderivatives involving the Power Rule is also quite simple.

Consider…

Ex. $f(x) = 3x^2$

$\boxed{F(x) = x^3 + C}$

$f(x) = 1x^4$

$F(x) = \frac{1}{5}x^5 + C$

$f(x) = 3x^4$

$F(x) = \frac{3}{5}x^5 + C$

Thus, the "Anti-Power Rule" ~~is as follows:~~

If $f(x) = bx^a$, then $F(x) = \boxed{\frac{b}{a+1}x^{a+1} + C}$

Example

$9x^0$

$\frac{9}{1}x^1$

Find the antiderivatives of the following:

a. $f(x) = x^2$

$F(x) = \frac{1}{3}x^3 + C$

b. $f(x) = 5x^3 - 8x^2 + 9$

$F(x) = \frac{5}{4}x^4 - \frac{8}{3}x^3 + 9x + C$

Differential Equations

Finding the antiderivatives can be presented in multiple ways. One of the ways is by way of differential equations.

Ex. Find the general solution of the differential equation $y' = 2$

$y = 2x + c$

Example

Find the particular solution of $f(x) = \frac{1}{x^2}$, with $x > 0$ and the initial condition $F(1) = 0$.

x^{-2}

$F(x) = \frac{1}{-1}x^{-1} + C$

$= -\frac{1}{x} + C$

$F(1) = \frac{-1}{1} + C = 0$

$= -1 + C = 0$

$C = 1$

$$\boxed{F(x) = -\frac{1}{x} + 1}$$

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