Prove the following Identities.

6. $\sin^2 x\cot^2 x + \cos^2 x\tan^2 x = 1$

LHS $= \frac{\cancel{\sin^2 x}}{1}\cdot\frac{\cos^2 x}{\cancel{\sin^2 x}} + \frac{\cancel{\cos^2 x}}{1}\cdot\frac{\sin^2 x}{\cancel{\cos^2 x}}$

$= \cos^2 x + \sin^2 x = 1 = RHS$

7. FOIL $(\tan x + \cot x)^2 = \csc^2 x\sec^2 x$

LHS $= \tan^2 x + 2\tan x\cot x + \cot^2 x$

$= \tan^2 x + 2(1) + \cot^2 x = \tan^2 x + 1 + 1 + \cot^2 x$

$= (\tan^2 x + 1) + (1 + \cot^2 x) = (\sec^2 x) + (\csc^2 x)$.

$= \frac{1}{\cos^2 x} + \frac{1}{\sin^2 x} = \frac{\sin^2 x + \cos^2 x}{\cos^2 x\sin^2 x} = \frac{1}{\cos^2 x\sin^2 x}$.

$= \boxed{\csc^2 x\sec^2 x} = RHS$

8. $(1-\tan x)(1-\cot x) = 2 - \sec x\csc x$

$= 1 - \cot x - \tan x + \tan x\cot x$

$= 1 - \cot x - \tan x + 1 = 2 - \cot x - \tan x$

$= 2 - \frac{\cos x}{\sin x} - \frac{\sin x}{\cos x} = \frac{2}{1} - \left(\frac{\cos^2 x}{\sin x\cos x} - \frac{\sin^2 x}{\sin x\cos x}\right)$

$= 2 - \frac{\cos^2 x}{\sin x\cos x} + \frac{\sin^2 x}{\sin x\cos x} = 2 - \frac{1}{\sin x\cos x} = 2 - \sec x\csc x = RHS$

9. $\sin\theta(\cot\theta + \tan\theta) = \sec\theta$

LHS $= \sin\theta\cot\theta + \sin\theta\tan\theta =$

$= \frac{\cancel{\sin\theta}}{1}\cdot\frac{\cos\theta}{\cancel{\sin\theta}} + \frac{\sin\theta}{1}\cdot\frac{\sin\theta}{\cos\theta} = \frac{\cos\theta}{1} + \frac{\sin^2\theta}{\cos\theta}$

$= \frac{\cos^2\theta}{\cos\theta} + \frac{\sin^2\theta}{\cos\theta} = \frac{\cos^2\theta + \sin^2\theta}{\cos\theta} = \frac{1}{\cos\theta}$

$= \sec\theta = RHS$

10. $\frac{\cos 3x - \cos 7x}{\sin 3x + \sin 7x} = \tan 2x$

odd

LHS $= \frac{-2\sin\left(\frac{3x+7x}{2}\right)\sin\left(\frac{3x-7x}{2}\right)}{2\sin\left(\frac{3x+7x}{2}\right)\cos\left(\frac{3x-7x}{2}\right)} = \frac{-2\cancel{\sin 5x}\sin(-2x)}{2\cancel{\sin 5x}\cos(-2x)}$

even

$= \frac{-(-\sin 2x)}{\cos 2x} = \frac{\sin 2x}{\cos 2x} = \tan 2x = RHS$

11. $\frac{\sin 2x}{\sin x} - \frac{\cos 2x}{\cos x} = \sec x$

LHS $= \frac{2\sin x\cos x}{\sin x} - \frac{2\cos^2 x - 1}{\cos x}$

$= \frac{2\sin x\cos x\cdot\cos x}{\sin x\cos x} - \frac{(2\cos^2 x\sin x - \sin x)}{\sin x\cos x}$

$= \frac{\cancel{2\sin x\cos^2 x}\ \cancel{-2\cos^2 x\sin x} + \sin x}{\sin x\cos x} = \frac{\cancel{\sin x}}{\cancel{\sin x}\cos x}$

$= \frac{1}{\cos x} = \sec x = RHS$

12. $4(\sin^6 x + \cos^6 x) = 4 - 3\sin^2 2x$

Note: $\sin^6 x = (\sin^2 x)(\sin^2 x)(\sin^2 x)$

LHS $= 4((\sin^2 x)(\sin^2 x)(\sin^2 x) + \cos^6 x)$

$= 4((1-\cos^2 x)(1-\cos^2 x)(1-\cos^2 x) + \cos^6 x)$

$= 4((1 - 2\cos^2 x + \cos^4 x)(1-\cos^2 x) + \cos^6 x)$

$= 4(1 - \cos^2 x - 2\cos^2 x + 2\cos^4 x + \cos^4 x \cancel{-\cos^6 x} \cancel{+\cos^6 x})$

$= \boxed{4(1 - 3\cos^2 x + 3\cos^4 x)}$

RHS $= 4 - 3(\sin 2x)(\sin 2x)$

$= 4 - 3(2\sin x\cos x)(2\sin x\cos x) = 4 - 3\overset{12}{(4\sin^2 x\cos^2 x)}$

$= 4(1 - 3\sin^2 x\cos^2 x)$

$= 4(1 - 3(1-\cos^2 x)\cos^2 x)$

$= 4(1 - 3(\cos^2 x - \cos^4 x)) = \boxed{4(1 - 3\cos^2 x + 3\cos^4 x)}$

Name: Key Period: ____ Date: ________

PreCalculus CH 7 Practice Test

Answer the following questions.

1. Simplify the following expressions.

a. $\frac{\cos x}{\sec x+\tan x}$

$= \frac{\cos x}{\frac{1}{\cos x} + \frac{\sin x}{\cos x}} = \frac{\cos x}{\frac{1+\sin x}{\cos x}} = \boxed{\frac{\cos^2 x}{1+\sin x}}$

b. $\frac{\tan x}{\sec(-x)}$ even function

$= \frac{\tan x}{\sec x} = \frac{\frac{\sin x}{\cancel{\cos x}}}{\frac{1}{\cancel{\cos x}}} = \boxed{\sin x}$

c. $\cos^3 x + \sin^2 x \cos x$

$\cos x(\cos^2 x + \sin^2 x)$

$\cos x(1)$

$= \boxed{\cos x}$

2. Use the addition or subtraction formula to find the **exact** value of the expression.

a. sin 15° $\boxed{\frac{\sqrt{6}-\sqrt{2}}{4}}$

$\sin(45-30)$

$= \sin 45 \cos 30 - \cos 45 \sin 30$

$\frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \cdot \frac{1}{2} = \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}$

b. $\cos\left(\frac{17\pi}{12}\right)$ $\frac{4\pi}{12}\ \frac{\pi}{3}$ $\frac{\pi}{4}\ \frac{5\pi}{12}$

$= \cos\left(\frac{8\pi}{12} + \frac{9\pi}{12}\right) = \cos\left(\frac{2\pi}{3} + \frac{3\pi}{4}\right)$

$= \cos\frac{2\pi}{3}\cos\frac{3\pi}{4} - \sin\frac{2\pi}{3}\sin\frac{3\pi}{4}$

$= \left(-\frac{1}{2}\right)\left(-\frac{\sqrt{2}}{2}\right) - \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$

$= \frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4} = \boxed{\frac{\sqrt{2}-\sqrt{6}}{4}}$

c. $\sin\left(-\frac{5\pi}{12}\right)$

$= \sin\left(\frac{3\pi}{12} - \frac{8\pi}{12}\right) = \sin\left(\frac{\pi}{4} - \frac{2\pi}{3}\right)$

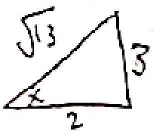
$= \sin\frac{\pi}{4}\cos\frac{2\pi}{3} - \sin\frac{2\pi}{3}\cos\frac{\pi}{4}$

$= \left(\frac{\sqrt{2}}{2}\right)\left(-\frac{1}{2}\right) - \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$

$= \frac{-\sqrt{2}}{4} - \frac{\sqrt{6}}{4} = \boxed{\frac{-\sqrt{2}-\sqrt{6}}{4}}$

3. Find $\sin 2x$, $\cos 2x$, and $\tan 2x$ from the given information.

$\tan x = 3/2 \Rightarrow$ $\cot x = \frac{2}{3}$, $\sin x > 0$ Soh Cah Toa



$\sin x = \frac{3}{\sqrt{13}} = \frac{3\sqrt{13}}{13}$

$\cos x = \frac{2}{\sqrt{13}} = \frac{2\sqrt{13}}{13}$

$\sin 2x = 2\sin x\cos x$
$= 2\left(\frac{3\sqrt{13}}{13}\right)\left(\frac{2\sqrt{13}}{13}\right)$
$= \frac{12 \cdot 13}{169} = \boxed{\frac{156}{169}}$

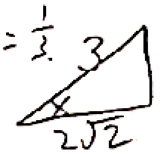
$\cos 2x = 1 - 2\sin^2 x$
$= 1 - 2\left(\frac{3\sqrt{13}}{13}\right)^2$
$= 1 - 2\left(\frac{9 \cdot 13}{169}\right)$
$= \frac{1}{1} - \frac{234}{169} = \boxed{\frac{-65}{169}}$

$\tan 2x = \frac{\sin 2x}{\cos 2x}$
$= \frac{156}{169} \cdot \frac{-169}{65}$
$= \boxed{\frac{-156}{65}}$

4. Find $\sin\frac{x}{2}$, $\cos\frac{x}{2}$, and $\tan\frac{x}{2}$ from the given information.

$\frac{1}{\sin x} = 3$. $\csc x = 3$, $90° < x < 180° \Rightarrow$ II quad

$\sin x = \frac{1}{3}$



$\sin x = \frac{1}{3}$

$\cos x = \frac{2\sqrt{2}}{3}$

$\sin\frac{x}{2} = \sqrt{\frac{1-\cos x}{2}} = \sqrt{\frac{1-\frac{2\sqrt{2}}{3}}{2}} = \sqrt{\frac{\frac{1-2\sqrt{2}}{3}}{2}} = \boxed{\sqrt{\frac{3-2\sqrt{2}}{6}}}$

$\cos\frac{x}{2} = \sqrt{\frac{1+\cos x}{2}} = \sqrt{\frac{1+\frac{2\sqrt{2}}{3}}{2}} = \boxed{\sqrt{\frac{3+2\sqrt{2}}{6}}}$

$\tan\frac{x}{2} = \frac{\sin x}{1+\cos x} = \frac{1/3}{1+\frac{2\sqrt{2}}{3}} = \frac{1}{\cancel{3}} \cdot \frac{\cancel{3}}{3+2\sqrt{2}} = \boxed{\frac{1}{3+2\sqrt{2}}}$

5. Use the half-angle formula to find the **exact** value of the expression.

a. cos 15°

$= \cos\left(\frac{30}{2}\right) = \sqrt{\frac{1+\cos(30)}{2}}$

$= \sqrt{\frac{1+\sqrt{3}/2}{2}} = \sqrt{\frac{2+\sqrt{3}}{2} \cdot \frac{1}{2}} = \sqrt{\frac{2+\sqrt{3}}{4}} = \boxed{\frac{\sqrt{2+\sqrt{3}}}{2}}$

b. $\sin\frac{11\pi}{12}$

$= \sin\left(\frac{\frac{22\pi}{12}}{2}\right) = \sin\left(\frac{11\pi/6}{2}\right)$

$= \sqrt{\frac{1-\cos 11\pi/6}{2}} = \sqrt{\frac{1-\left(\frac{\sqrt{3}}{2}\right)}{2}} = \sqrt{\frac{\frac{2+\sqrt{3}}{2}}{2}}$

$= \sqrt{\frac{2+\sqrt{3}}{4}} = \boxed{\frac{\sqrt{2+\sqrt{3}}}{2}}$