Name: Key **Period:** ______ **Date:** __________

PreCalculus CH 5 Practice Test

Answer the following questions.

1. What does it mean for a function to have a **period** of 2π?

The length of each cycle (repetition) is 2π.

2. Consider a soundwave. If the volume is to be increased, what changes? If the note or the pitch is to be changed, what in the equation changes?

Volume: amplitude is ~~i~~ changed. ; Pitch: "ω" changes.

3. Find the period and the amplitude (if applicable) of the following functions.

a. $f(t) = 4\sin t$

$a=4,\ k=1$

Per: $\frac{2\pi}{k} \Rightarrow \frac{2\pi}{1} = \boxed{2\pi}$

Amp: $|4| = \boxed{4}$

b. $f(t) = \frac{2}{3}\cos 3t$

$a=2/3 \quad k=3$

Per: $\boxed{\frac{2\pi}{3}}$

Amp: $|2/3| = \boxed{2/3}$

c. $f(t) = \tan 7\pi$

$a=1 \quad k=7$.

Per = ~~X~~ $1/7$

Amp: $|1| = \boxed{1}$

d. $f(t) = \csc\frac{16}{8}\pi(t-9)$

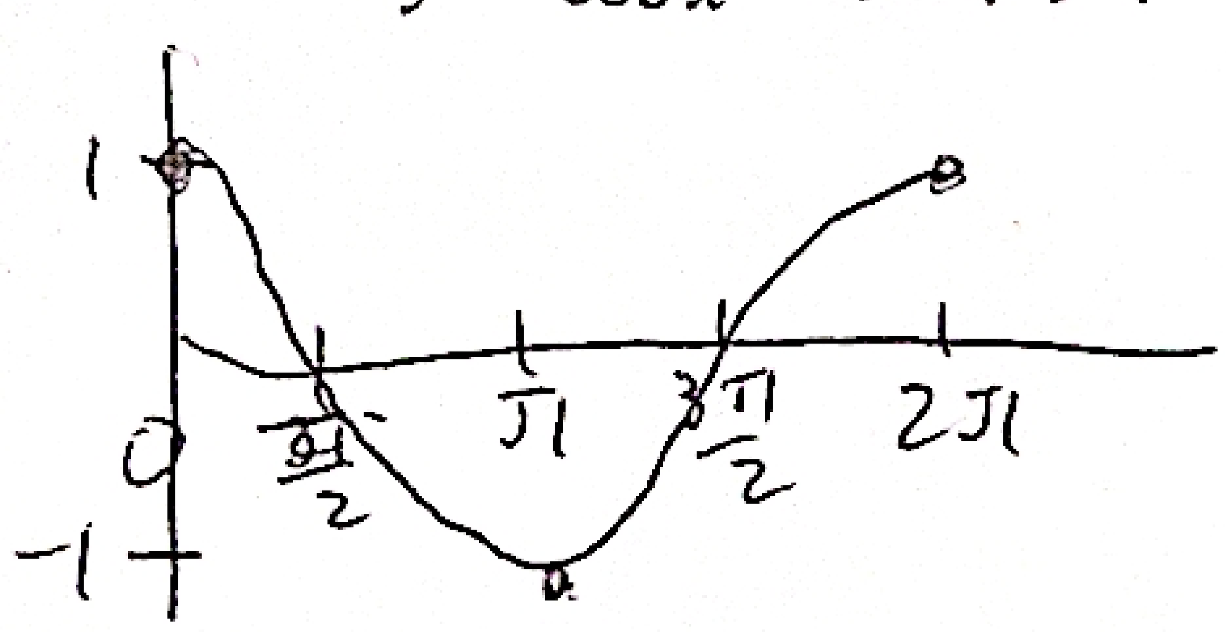
$a=1 \quad k=\frac{16\pi}{8}$

per: $\frac{2\pi}{16\pi/8} = \frac{2\pi}{1} \cdot \frac{8}{16\pi} = \boxed{1}$

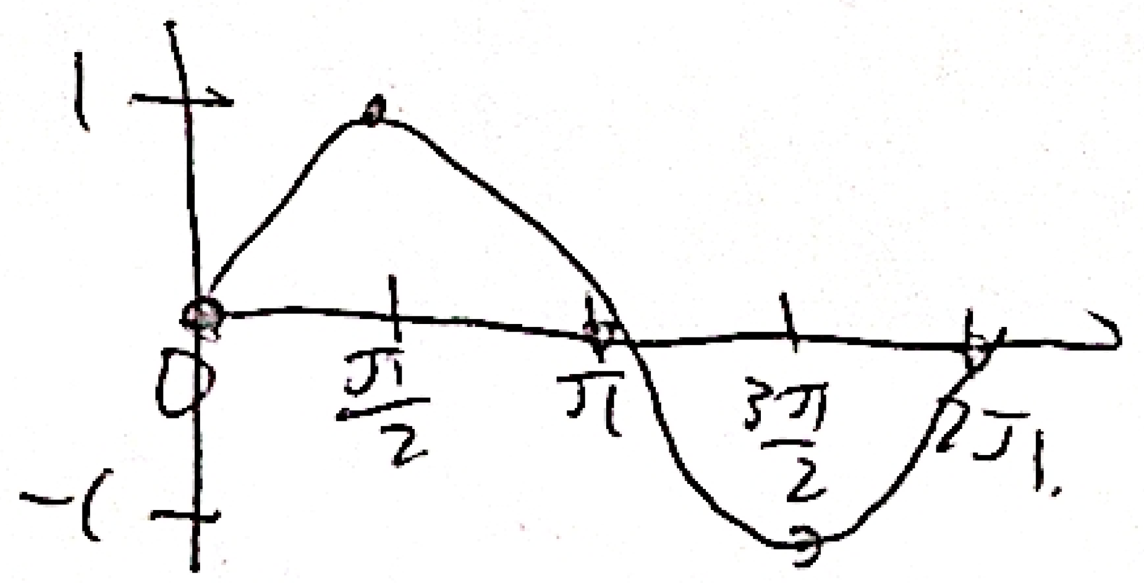
Amp: $|1| = \boxed{1}$.

4. Sketch the graph of the standard cosine and sine functions.

a. $y = \cos x$ amp: 1 per: 2π.

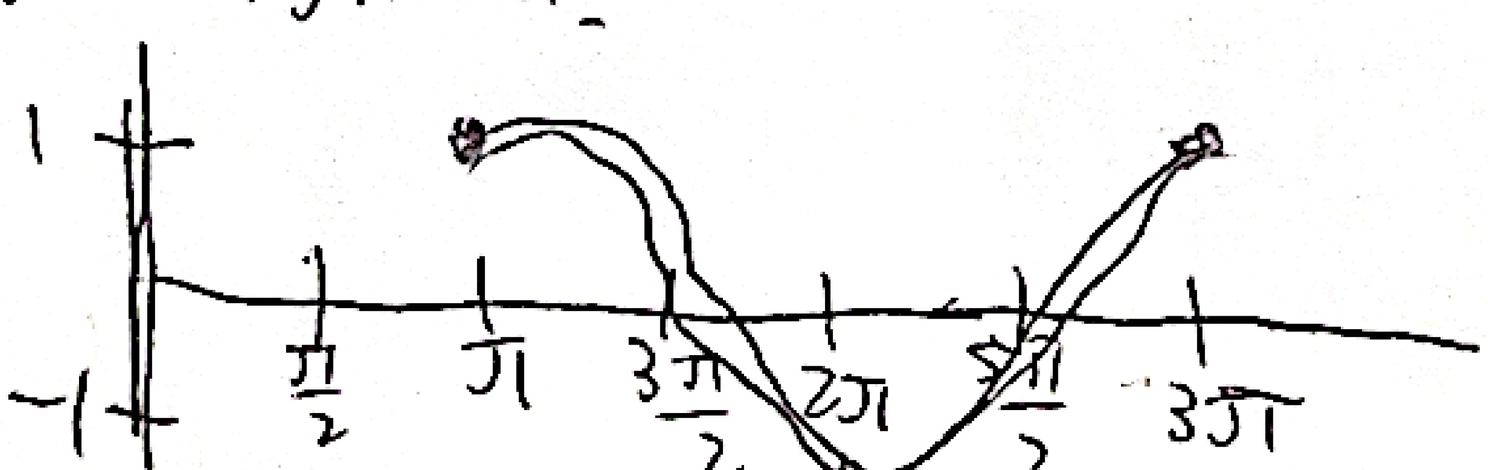


b. $y = \sin x$ amp = 1 per. = 2π.



5. Now, graph $y = \cos(x-\pi)$, **without** having to make a table, but simply by transforming the graph of $y = \cos x$ from #4.

Shift: right π.



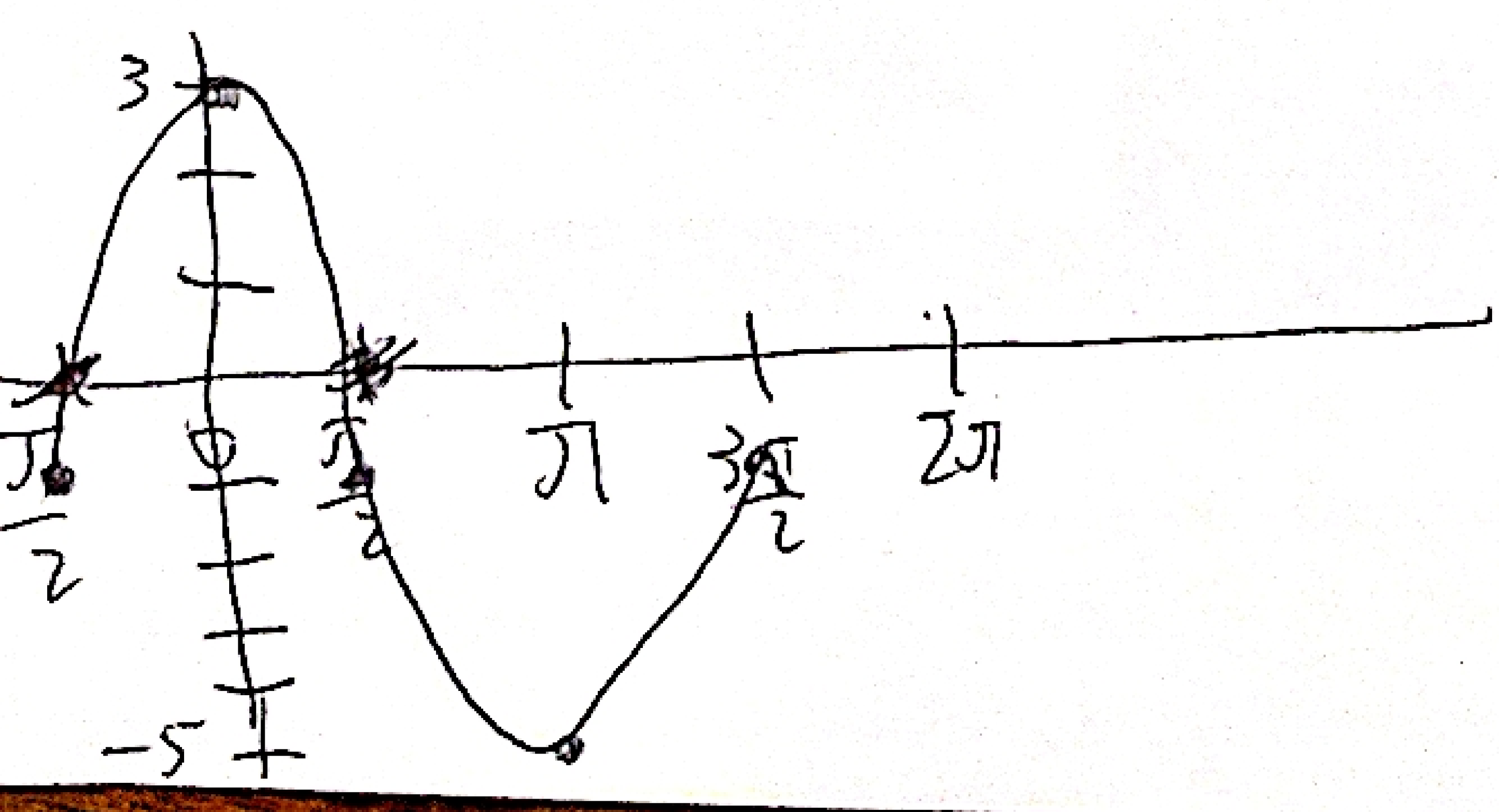
6. Sketch the following graph: $y = 4\sin\left(\pi + \frac{\pi}{2}\right) - 1$

Shift: left $\pi/2$, down 1

Amp: $|4| = 4$

h-pt: 4 → 3

l-pt: -4 → -5.



7. Evaluate the following trigonometric functions. (No calculators!)

III a. $\sin\frac{5\pi}{3}$ $\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$

$\boxed{-\frac{\sqrt{3}}{2}}$

b. $\sin\frac{15\pi}{4} = 2\frac{7\pi}{4} = 2\pi + \frac{7\pi}{4}$ $\left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$

So, same as $\sin\frac{7\pi}{4}$ (Quadrant IV)

$\boxed{-\frac{\sqrt{2}}{2}}$

c. $\cos\left(\cancel{2\pi} + \frac{5\pi}{6}\right)$ II $\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

$\boxed{-\frac{\sqrt{3}}{2}}$

II d. $\tan\frac{3\pi}{4}$ $\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

$\frac{\sin}{\cos} \Rightarrow \frac{y}{x} \Rightarrow \frac{\frac{\sqrt{2}}{2}}{-\frac{\sqrt{2}}{2}} = \boxed{-1}$

I e. $\sec\frac{\pi}{3} = \frac{1}{\cos\left(\frac{\pi}{3}\right)} = \frac{1}{x}$ $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

$\frac{1}{1/2} = \boxed{2}$

8. If $\sin t = -\frac{4}{5}$ and t is in quadrant III, find the values of all other trig functions at t.

$\cos t = \pm\sqrt{1-\sin^2 t} \Rightarrow \pm\sqrt{1-\left(\frac{4}{5}\right)^2} = \pm\sqrt{1-\frac{16}{25}} = \pm\sqrt{\frac{9}{25}} = \pm\frac{3}{5}$ But since quad. III

$\boxed{\cos t = -3/5}$. $\tan t = \frac{\sin t}{\cos t} = \frac{-4/5}{-3/5} = -\frac{4}{5} \cdot \frac{-5}{3} = \boxed{4/3}$ $\boxed{\cot t = \frac{3}{4}}$

$\boxed{\csc t = -5/4}$ $\boxed{\sec t = -5/3}$

9. Write the Pythagorean Identity, and use it to express $\sin t$ in terms of $\cos t$ and vice-versa.

Pythagorean Identity: $\sin t = \pm\sqrt{1-\cos^2 t}$ $\cos t = \pm\sqrt{1-\sin^2 t}$

$\sin^2 t + \cos^2 t = 1$

10. Use the various identities to write the first trigonometric expression in terms of the second, with terminal given by the quadrant:

a. $\sin t, \cos t$; quadrant I

$\boxed{\sin t = \sqrt{1-\cos^2 t}}$

b. $\tan t, \sin t$; quadrant IV

$\tan t = \frac{\sin t}{\cos t} = \boxed{\frac{\sin t}{+\sqrt{1-\sin^2 t}}}$

c. $\tan^2 t, \sin t$; any quadrant

$\tan^2 t = \frac{\sin^2 t}{\cos^2 t}$.

$= \boxed{\frac{\sin^2 t}{1-\sin^2 t}}$.

d. $\csc t, \cot t$; any quadrant

$\csc^2 t = 1 + \cot^2 t$.

$\boxed{\csc t = \pm\sqrt{1+\cos^2 t}}$

11. Find the amplitude, period, and frequency of each motion.

a. $y = 2\sin 3t$

amp: $|a| = |2| = \boxed{2}$

per: $\frac{2\pi}{\omega} = \boxed{\frac{2\pi}{3}}$

b. $y = 3\cos\frac{1}{2}t$

amp: $|3| = \boxed{3}$

per: $\frac{2\pi}{1/2} = \boxed{4\pi}$

c. $y = 1.6\sin(t - 1.8)$

amp: $|1.6| = \boxed{1.6}$

per: $\frac{2\pi}{1} = \boxed{2\pi}$

d. $y = -\cos 0.3t$

amp: $|-1| = \boxed{1}$

per: $\frac{2\pi}{0.3} = \frac{2\pi}{3/10} = \boxed{\frac{20\pi}{3}}$

e. $y = 2.43\sin 3.6t$

amp: $|2.43| = \boxed{2.43}$

Per: $\frac{2\pi}{3.6} = \frac{2\pi}{3\,6/10} = \frac{2\pi}{18/5} = \frac{10\pi}{18} = \boxed{\frac{5\pi}{9}}$

$\boxed{3\,3/5 = \frac{18}{5}}$

f. $y = 5\cos(\frac{2}{3}t + \frac{3}{4})$

amp: $|5| = \boxed{5}$

per: $\frac{2\pi}{2/3} = \frac{6\pi}{2} = \boxed{3\pi}$

12. Find a function that models the simple harmonic motion having the given properties. Assume that the displacement is **zero** at time $t = 0$. → $\boxed{a \sin \omega t}$

a. amplitude: 10
period: $\frac{1}{2}$

$a = \boxed{10}$

$\omega \Rightarrow \frac{1}{2} = \frac{2\pi}{\omega}$

$\Rightarrow \omega = \boxed{4\pi}$

$\boxed{y = 10\sin 4\pi t}$

b. amplitude: 5
frequency: 1

$a = \boxed{5}$

$\omega \Rightarrow \frac{\omega}{2\pi} = 1$

$\omega = \boxed{2\pi}$

$\boxed{y = 5\sin 2\pi t}$

c. amplitude: 0.2
period: 40π

$a = 0.2$

$\omega \Rightarrow \frac{2\pi}{\omega} = 40\pi$

$\omega = \boxed{\frac{1}{20}}$

$\boxed{y = 0.2\sin\frac{1}{20}t}$

13. Find a function that models the simple harmonic motion having the given properties. Assume that the displacement is at its **maximum** at time $t = 0$. → $y = a\cos\omega t$.

a. amplitude: 135
period: $\frac{1}{70}$

$a = |135| = 135$

$\omega \Rightarrow$ per $= \frac{2\pi}{1/70} = 140\pi$

$\boxed{y = 135\cos 140\pi t}$

b. amplitude: 155
frequency: 60

$a = 155$

$\omega \Rightarrow \frac{\omega}{2\pi} = 60$

$\omega = 120\pi$

$\boxed{y = 155\cos 120\pi t}$

c. amplitude: 0.35
frequency: $\frac{1}{4}\pi$

$a = 0.35$

$\omega \Rightarrow \frac{\omega}{2\pi} = \frac{1}{4}\pi$

$\omega = \frac{1}{2}$

$\boxed{y = 0.35\cos\frac{1}{2}t}$

14. In a predator/prey model, the predator population is modeled by the function:

$$y = 900\cos 2t + 8000$$

What is the **maximum** population?

Cos function starts at the maximum (highest pt.). So,

$900\cos 2(0) + 8000 = 900 + 8000 = \boxed{8900}$

$\cos 0 = 1$

15. Each time your heart beats, your blood pressure increases, then decreases as the heart rests between beats. A certain person's blood pressure is modeled by the function:

$$p(t) = 115 + 25\sin(160\pi t)$$

$a = 25$, $\omega = 160\pi$

Find the amplitude, period, and frequency of p.

$amp = |25| = \boxed{25}$

$per = \frac{2\pi}{\omega} = \frac{2\pi}{160\pi} = \boxed{\frac{1}{80}}$

$Freq = \boxed{80}$

16. The variation of the water level relative to mean sea level in Commencement Bay at Tacoma, Washington, for a particular 24-hour period, starts at mean sea level, rises to 6ft, drops 6ft below, then returns to mean sea level. Assuming that this motion is simple harmonic:

Sin

a. Find an equation that describes the height of the tide in Commencement Bay.

Per = 24 hrs

$a = 6$

$\omega \Rightarrow 24 = \frac{2\pi}{\omega}$

$\omega = \frac{\pi}{12}$

$\boxed{y = 6\sin\frac{\pi}{12}t}$

b. What is the water level at 6pm?

6:00pm = 18:00 so, $t = 18$

$y = 6\sin\frac{\pi}{12}(18) = 6\sin\frac{3\pi}{2}$

$= 6(-1) = \boxed{-6ft}$