Calculus BC Name____________________ ID: 1

Chapter 3 Exam Review II

Date________________ Period____

For each problem, find the: x-coordinates of the critical points, open intervals where the function is increasing and decreasing, x-coordinates of the inflection points, open intervals where the function is concave up and concave down, and relative minima and maxima. Using this information, sketch the graph of the function.

1) $f(x) = \frac{x^3}{3} + x^2$

2) $f(x) = \frac{2}{x^2 - 9}$

3) $y = -\frac{x^2}{2} + x + \frac{5}{2}$

4) $y = -\frac{3x}{x + 1}$

5) $y = \frac{3x}{x - 3}$

6) $y = -x^2 - 2x - 1$

Solve each optimization problem.

7) A rancher wants to construct two identical rectangular corrals using 200 ft of fencing. The rancher decides to build them adjacent to each other, so they share fencing on one side. What dimensions should the rancher use to construct each corral so that together, they will enclose the largest possible area?

8) An architect is designing a composite window by attaching a semicircular window on top of a rectangular window, so the diameter of the top window is equal to and aligned with the width of the bottom window. If the architect wants the perimeter of the composite window to be 18 ft, what dimensions should the bottom window be in order to create the composite window with the largest area?

For each problem, find a linear approximation of the given quantity.

9) $\cos 1361°$

10) $\sqrt{25.3}$

11) 0.99^3

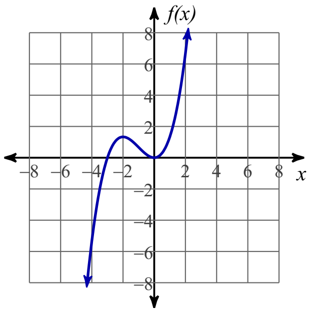
12) $\cos 1191°$

Use differentials to solve each problem.

13) The sides of a cube are measured to be 8 in, with a possible error of $\pm\frac{1}{5}$ in. Estimate the possible propagated error in the calculated volume.

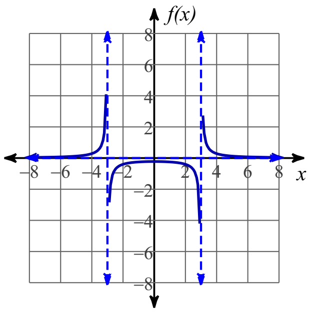
Answers to Chapter 3 Exam Review II (ID: 1)

1)



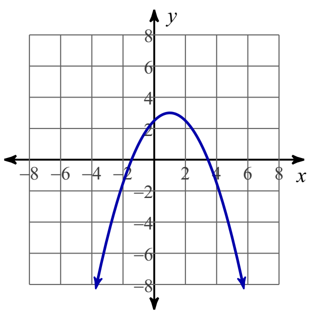
Critical points at: $x = -2, 0$
Increasing: $(-\infty, -2), (0, \infty)$ Decreasing: $(-2, 0)$
Inflection point at: $x = -1$
Concave up: $(-1, \infty)$ Concave down: $(-\infty, -1)$
Relative minimum: $(0, 0)$ Relative maximum: $\left(-2, \frac{4}{3}\right)$

2)



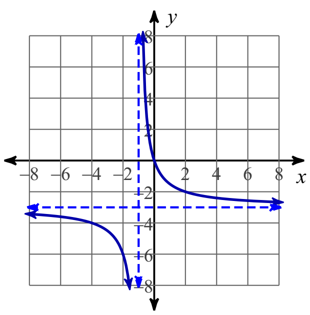
Critical point at: $x = 0$
Increasing: $(-\infty, -3), (-3, 0)$ Decreasing: $(0, 3), (3, \infty)$
No inflection points exist.
Concave up: $(-\infty, -3), (3, \infty)$ Concave down: $(-3, 3)$
No relative minima. Relative maximum: $\left(0, -\frac{2}{9}\right)$

3)



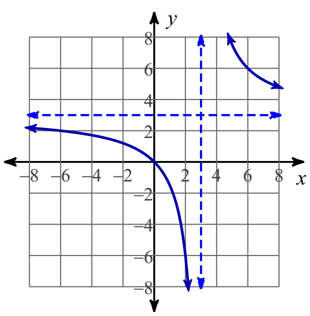
Critical point at: $x = 1$
Increasing: $(-\infty, 1)$ Decreasing: $(1, \infty)$
No inflection points exist.
Concave up: No intervals exist. Concave down: $(-\infty, \infty)$
No relative minima. Relative maximum: $(1, 3)$

4)



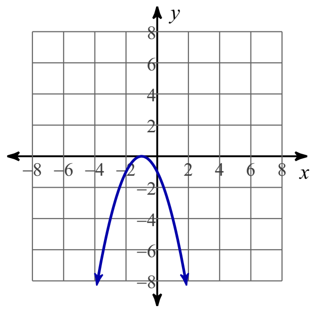
No critical points exist.
Increasing: No intervals exist. Decreasing: $(-\infty, -1), (-1, \infty)$
No inflection points exist.
Concave up: $(-1, \infty)$ Concave down: $(-\infty, -1)$
No relative minima. No relative maxima.

5)



No critical points exist.
Increasing: No intervals exist. Decreasing: $(-\infty, 3), (3, \infty)$
No inflection points exist.
Concave up: $(3, \infty)$ Concave down: $(-\infty, 3)$
No relative minima. No relative maxima.

6)



Critical point at: $x = -1$
Increasing: $(-\infty, -1)$ Decreasing: $(-1, \infty)$
No inflection points exist.
Concave up: No intervals exist. Concave down: $(-\infty, \infty)$
No relative minima. Relative maximum: $(-1, 0)$

7) 25 ft (non-adjacent sides) by $\frac{100}{3}$ ft (adjacent sides)

8) $\frac{36}{4 + \pi}$ ft (width) by $\frac{18}{4 + \pi}$ ft (height)

9) $f(x) = \cos x,\ f'(x) = -\sin x$

$x_0 = \frac{3\pi}{4}$ radians, $\Delta x = \frac{\pi}{180}$ radians

$$f(x_0 + \Delta x) \approx f(x_0) + f'(x_0)\Delta x = \frac{\sqrt{2}(-180 - \pi)}{360} \approx -0.7194$$

10) $f(x) = \sqrt{x},\ f'(x) = \frac{1}{2x^{\frac{1}{2}}}$

$x_0 = 25,\ \Delta x = 0.3$

$$f(x_0 + \Delta x) \approx f(x_0) + f'(x_0)\Delta x = \frac{503}{100} = 5.03$$

11) $f(x) = x^3,\ f'(x) = 3x^2$

$x_0 = 1,\ \Delta x = -0.01$

$$f(x_0 + \Delta x) \approx f(x_0) + f'(x_0)\Delta x = \frac{97}{100} = 0.97$$

12) $f(x) = \cos x,\ f'(x) = -\sin x$

$x_0 = \frac{2\pi}{3}$ radians, $\Delta x = -\frac{\pi}{180}$ radians

$$f(x_0 + \Delta x) \approx f(x_0) + f'(x_0)\Delta x = \frac{-180 + \pi\sqrt{3}}{360} \approx -0.4849$$

13) $\pm\frac{192}{5} = \pm 38.4 \text{ in}^3$

