Name: Key **Period:** ______ **Date:** ___________

Calculus BC Chapter 3 Review

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Answer the following questions. Show all work! No calculators allowed!!

(2 pts.) 1. Explain in words how to find critical values.

Find derivative, solve for the zeros.

(2 pts. Each)

2. (T or [✓ F]) Every critical value is a relative extrema.
3. (T or [✓ F]) An absolute extremum on a closed interval must be one of the critical values.
4. ([✓ T] or F) Every relative extremum must be one of the critical values.
5. ([✓ T] or F) First derivative describes increasing or decreasing behavior, while the second derivative describes the concavity.

(5 pts. each)

6. Find the extrema of ~~y~~ $f(x)$ $= 2x - 3x^{\frac{2}{3}}$ on the interval $[-1, 3]$.

$y' = 2 - 2x^{-1/3}$

$0 = 2 - 2x^{-1/3} \Rightarrow 0 = 1 - x^{-1/3}$

$0 = \frac{x^{1/3} - 1}{x^{1/3}} \Rightarrow$ CV= $\boxed{x = 1}$

dis @ $\boxed{x=0}$

$f(-1) \to -2 - 3 = \boxed{-5}$ $\boxed{\text{min}}$

$f(0) \to \boxed{0}$ max

$f(1) \to 2 - 3 = -1$

$f(3) \to 6 - 3\sqrt[3]{9}$ -5

7. Can the Mean Value Theorem be applied to $f(x) = \frac{x+1}{x}$ in the interval $[\frac{1}{2}, 2]$? Explain your answer!

$f(2) = 3/2$

$f(1/2) = 3$

$\frac{3 - 3/2}{1/2 - 2} = \frac{3/2}{-3/2} = -1$

$f'(x) = \frac{x - (x+1)}{x^2} = \frac{\cancel{x - x} - 1}{x^2} = \frac{-1}{x^2}$

$-\frac{1}{x^2} = -1$ $x = 1$ and $1/2 < x < 2$ ✓

$\boxed{\text{Yes}}$

(7 pts. each)

Sketch the curve. Be sure to include all information!

8. $f(x) = \frac{x}{x^2+1}$

(8 pts.)

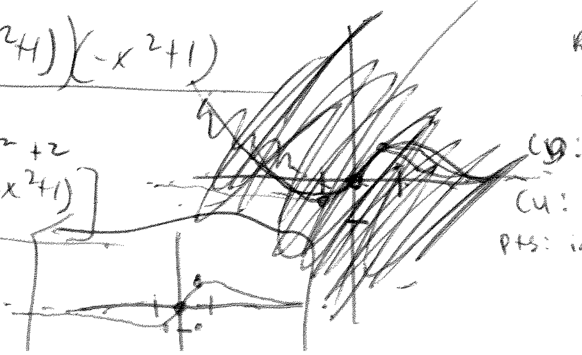
$f'(x) = \frac{x^2+1 - 2x(x)}{(x^2+1)^2} = \boxed{\frac{-x^2+1}{(x^2+1)^2}}$

$f''(x) = \frac{-2x(x^2+1)^2 - (2(x^2+1) \cdot 2x)(-x^2+1)}{(x^2+1)^4}$

$= \frac{-2x(x^2+1)^2 - (4x(x^2+1))(-x^2+1)}{(x^2+1)^4}$

$\frac{-2x(x^2+1)[(x^2+1) + 2(-x^2+1)]}{(x^2+1)^4}$ $\quad -2x^2+2$

$\boxed{\frac{x(x^2+1)(-x^2+3)}{(x^2+1)^4}}$



x-int: (0, 0)

y-int: (0, 0)

VA: DNE

HA: y=0

CV: ±1

$$\begin{array}{c|c|c} & -1 & 1 \\ \hline - & + & - \end{array}$$

$$\begin{array}{c|c|c|c} & -\sqrt{3} & 0 & \sqrt{3} \\ \hline - & + & - & + \end{array}$$

Inc: (-1, 1) Dec: (-∞, -1) ∪ (1, ∞)

Rel min @ (-1, -1/2), max @ (1, 1/2)

f'' CV: x = 0, ±√3

CD: (-∞, -√3) ∪ (0, √3)

CU: (-√3, 0) ∪ (√3, ∞)

Pts: inf @ (-√3, -√3/4), (0, 0), (√3, √3/4)

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$\overset{0}{+} \mid - \mid \overset{4}{+}$

(8 pts.) 9. $f(x) = x + \frac{32}{x^2} = \frac{x^3+32}{x^2}$

$f'(x) = \frac{3x^2(x^2) - 2x(x^3+32)}{x^4}$

$= \boxed{\frac{x^4 - 64x}{x^4}}$ f

$f''(x) = \frac{(4x^3-64)(x^4) - 4x^3(x^4-64x)}{x^8}$

$= \frac{-64x^4 + 256x^4}{x^8}$

$= \boxed{\frac{192x^4}{x^8}}$ +

x-int: $(\sqrt[3]{-32}, 0)$

y-int: DNE

VA: x=0.

HA: DNE

f' CV: ~~0~~, 4

Inc: $(-\infty, 0) \cup (4, \infty)$

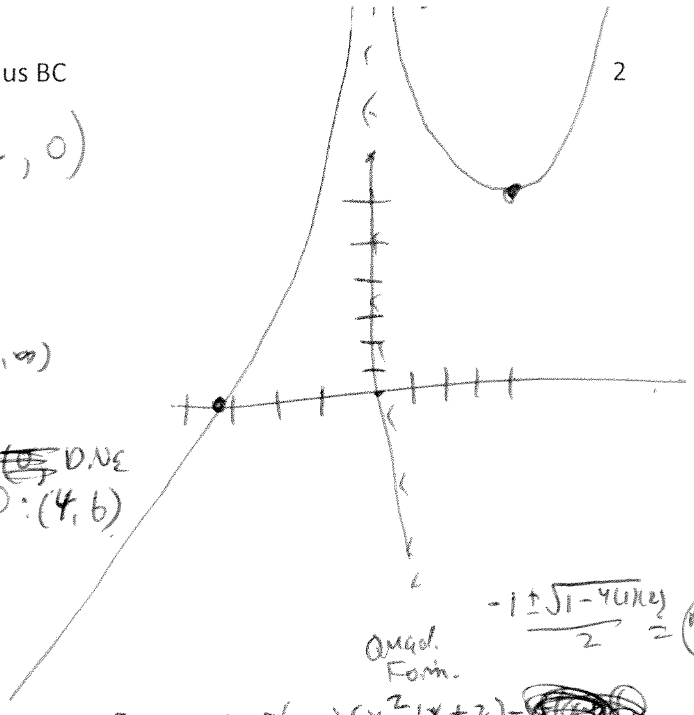
Dec: (0, 4).

Rel max @ ~~(0,~~ DNE

Rel min @: (4, 6)

f'' CV: DNE

$\overset{0}{+ \mid +}$



$\frac{-1 \pm \sqrt{1-4(1)(2)}}{2}$ = No Sol.

Quad. Form.

(8 pts.) 10. $y = 2 - x - x^3$

$y' = -1 - 3x^2$

$y'' = -6x$

1	-1	0	-1	2
+	↓	-1	-1	-2
	-1	-1	-2	0

$y = (x-1)(-x^2-x-2) = -(x-1)(x^2+x+2) =$

x = 1

x-int: (1, 0)

y-int: (0, 2)

VA: DNE

HA: DNE

CV: $\pm\sqrt{1/3}$

$-\sqrt{1/3}$ $\sqrt{1/3}$

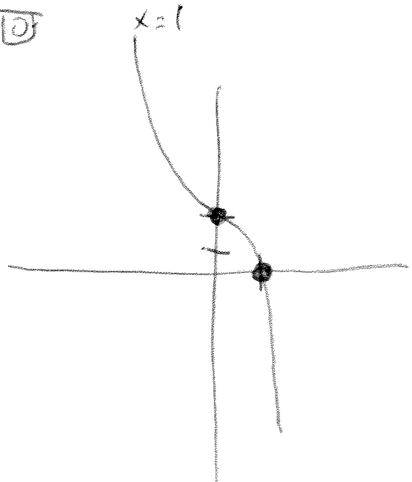
$- \mid - \mid -$

Decreasing $(-\infty, \infty)$

f'' CV: 0.

$\overset{0}{\underset{cu}{+} \mid \underset{cd}{-}}$

pt. of infl.: (0, 2)



11. $g(x) = x\sqrt{9-x} = x(9-x)^{1/2}$

$g'(x) = (9-x)^{1/2} + \frac{1}{2}(9-x)^{-1/2} \cdot (-1)(x)$

$= (9-x)^{1/2} - \frac{1}{2}x(9-x)^{-1/2}$

$g'(x) = \sqrt{9-x} - \frac{\frac{1}{2}x}{\sqrt{9-x}} = \frac{9-x-\frac{1}{2}x}{\sqrt{9-x}} = \boxed{\frac{9-\frac{3}{2}x}{\sqrt{9-x}}}$ +

x-int: (0, 0), (9, 0)

y-int: (0, 0)

VA: DNE

HA: DNE

Domain: $\boxed{x \le 9}$

g' CV: 6

Inc: $(-\infty, 6)$

Dec: $(6, \infty)$

Rel max @: $(6, 6\sqrt{3})$

g'' CV: -12

$\overset{6}{+ \mid -}$

$\overset{-12}{\underset{cu}{+} \mid \underset{c}{-}}$

$g''(x) = \frac{1}{2}(9-x)^{-1/2} \cdot -1 - \left(\frac{1}{2}(9-x)^{-1/2} + \left(-\frac{1}{2}(9-x)^{-3/2} \cdot -1\right)\left(\frac{1}{2}x\right)\right)$

$= -\frac{1}{2}(9-x)^{-1/2} - \left(\frac{1}{2}(9-x)^{-1/2} + \frac{1}{4}x(9-x)^{-3/2}\right)$

$= \boxed{-(9-x)^{-1/2} - \frac{1}{4}x(9-x)^{-3/2}}$

$= \frac{-1}{\sqrt{9-x}} - \frac{\frac{1}{4}x}{(\sqrt{9-x})^3} = \frac{-1}{\sqrt{9-x}} - \frac{\frac{1}{4}x}{(9-x)\sqrt{9-x}} = \frac{-9+x-\frac{1}{4}x}{(9-x)\sqrt{9-x}} \boxed{\frac{-9-\frac{3}{4}x}{(9-x)\sqrt{9-x}}}$