Name: Key **Period:** _____ **Date:** ___________

PreCalculus Chapter 2 Practice Test

Answer the following questions. No work is necessary unless it is specified.

1. Define function.

A relation in which for every input there is exactly one output.

2. For the following piecewise function, evaluate the function at the indicated values.

$$f(x) = \begin{cases} 5 & \text{if } x \le 2 \\ 2x - 3 & \text{if } x > 2 \end{cases}$$

$f(-3), f(0), f(2), f(3), f(5)$

$f(-3) = 5$

$f(0) = 5$

$f(2) = 5$

$f(3) = 3$

$f(5) = 7$.

3. For the function $f(x) = 2x^2 + 8x - 1$

a. Find its domain.

$(-\infty, \infty)$

b. Complete the square and write it in the vertex form: $f(x) = a(x-h) + k$

$\left(\frac{b}{2}\right)^2 = \left(\frac{4}{2}\right)^2 = (2)^2 = 4.$

$\frac{f(x)}{2} = x^2 + 4x - \frac{1}{2}$

$4 + \frac{f(x)}{2} = x^2 + 4x + 4 - \frac{1}{2}$

$4 + \frac{f(x)}{2} = (x+2)^2 - \frac{1}{2}$

$\frac{f(x)}{2} = 2\left[(x+2)^2 - \frac{9}{2}\right] \Rightarrow \boxed{f(x) = 2(x+2)^2 - 9}$

c. Find its vertex and determine whether it's a maximum or a minimum point.

$(h, k) \rightarrow \boxed{(-2, -9)}$ Minimum

d. Describe the graph's change (shift, stretch, compress, etc.) from $f(x) = x^2$ and graph the function.

Left 2, down 9, vertical stretch by factor of 2.

4. Find the domain of the following functions.

a. $f(x) = \frac{3}{x^2+3}$

$x^2+3=0$ $\boxed{(-\infty, \infty)}$

$x^2 = -3$

$x = \sqrt{-3} \Rightarrow$ No sol.

b. $f(x) = \sqrt{-3x-7}$ $\boxed{(-\infty, -7/3]}$

$-3x-7 \geq 0$

$-3x \geq 7$

$x \leq -7/3$

c. $h(x) = \frac{2}{\sqrt{7-6x}}$ $\boxed{(-\infty, 7/6]}$

$7-6x \geq 0$

$-6x \geq -7$

$x \leq 7/6$

5. Write the following equation for y in terms of x: $3x + 4y = 2$

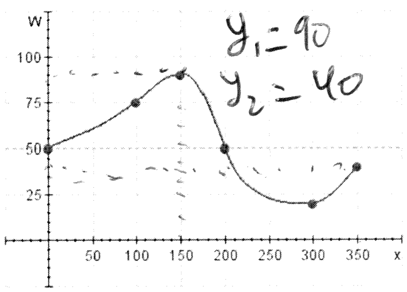
$4y = 2-3x$

$\boxed{y = \frac{1}{2} - 3/4x}$

6. Write the following equation for x in terms of y: $x - 2y - 3 = 0$

$\boxed{x = 2y+3}$

7. Use the graph to state the intervals in which the function is increasing, and decreasing. Then, find the average rate of change between $x_1 = 150$ and $x_2 = 350$.



$y_1 = 90$

$y_2 = 40$

Inc: $[0, 150]$, $[300, 350]$

Dec: $[150, 300]$

$ARC = \frac{40-90}{350-150} = \frac{-50}{200} = \boxed{-1/4}$

8. For the function $f(x) = 3x - 2$, determine the average rate of change between $x_1 = 2$, and $x_2 = 3$.

$y_1 = 4$ $y_2 = 7$

$ARC = \frac{7-4}{3-2} = \frac{3}{1} = \boxed{3}$

9. Determine whether the following functions are one-to-one. If they are, find their inverse function.

a. $f(x) = -2x + 4$

1) 1-1?

$f(x_1) = f(x_2)$

$-2x_1 + \cancel{4} = -2x_2 + \cancel{4}$

$\frac{-2x_1}{-2} = \frac{-2x_2}{-2}$

$x_1 = x_2$ ✓

2) Inverse

$y = -2x+4$

$x = -2y+4$

$\Rightarrow \boxed{y = \frac{x-4}{-2}}$

b. $f(x) = \sqrt{x}$

1) 1-1?

$f(x_1) = f(x_2)$

$(\sqrt{x_1})^2 = (\sqrt{x_2})^2$

$x_1 = x_2$ ✓

2) Inverse

$y = \sqrt{x}$

$x = \sqrt{y}$

$\boxed{y = x^2}$

c. $g(x) = x^2 - 2x$

Not

1-1

(even powered polynomial)

d. $h(x) = x^3 + 8$

$h(x_1) = h(x_2)$

$x_1^3 + \cancel{8} = x_2^3 + \cancel{8}$

$\sqrt[3]{x_1^3} = \sqrt[3]{x_2^3}$

$x_1 = x_2$ ✓

$y = x^3 + 8$

$x = y^3 + 8$

$-8 \quad -8$

$x - 8 = y^3$

$y = \sqrt[3]{x-8}$

$\boxed{f^{-1}(x) = \sqrt[3]{x-8}}$

10. Let $f(x) = x - 3$ and $g(x) = 4x^2$. Find $f + g$, $f - g$, fg, $\frac{f}{g}$, $f \circ g$, $g \circ f$

$f+g = x - 3 + 4x^2$ $\qquad \frac{f}{g} = \frac{x-3}{4x^2}$

$f-g = x - 3 - 4x^2$

$fg = (x-3)(4x^2) = 4x^3 - 12x^2$ $\qquad f \circ g = f(g(x)) = (4x^2) - 3 \quad \boxed{4x^2 - 3}$

$g \circ f = g(f(x)) = \boxed{4(x-3)^2}$

11. Use $f(x) = 3x - 5$ and $g(x) = 2 - x^2$ to evaluate the following expressions.

a. $(f \circ g)(0)$ b. $(f \circ g)(2)$ c. $(f \circ f)(3)$ d. $(g \circ f)(1)$

a. $f(g(0)) = f(2) = \boxed{1}$

b. $f(g(2))$, $f(-2) = 3(-2) - 5 = \boxed{-11}$

c. $f(f(3))$, $f(4) = 3(4) - 5 = \boxed{7}$

d. $g(f(1)) = g(-2)$, $2 - (-2)^2 = \boxed{-2}$

12. (T or F) Only one-to-one functions can have an inverse function.

13. (T or F) If a graph stretches vertically, then it also stretches horizontally.

14. (T or F) The set of all inputs (domain) of a function becomes the set of all outputs (range) for the inverse function.

15. (T or F) You can test for one-to-one-ness of a function using the vertical line test.

16. The effectiveness of a television commercial depends on how many times a viewer watches it. After some experiments an advertising agency found that if the effectiveness E is measured on a scale of 0 to 10, then $E(n) = \frac{2}{3}n - \frac{1}{90}n^2$, where n is the number of times a viewer watches a given commercial. For a commercial to have maximum effectiveness, how many times should a viewer watch it? (Use Calc.)

x value

Vertex!!!

$x: -\frac{b}{2a} = \frac{-2/3}{2(-1/90)} = -\frac{2}{3} \cdot \frac{-90}{2}$

$= \frac{-90}{3}$

$= \boxed{-30}$ times.

vertex!

17. A gardener has 240 feet of fencing to fence in a rectangular vegetable garden. Find the dimensions of the largest area she can fence. What is the maximum area?

y

P=240 x

$2x + 2y = 240$

$x + y = 120$

~~$x + 2y$~~

$y = 120 - x$

$A(x) = x(120 - x)$ ← (instead of xy).

$= 120x - x^2$

$x: \frac{-b}{2a} = \frac{-120}{-2}$

$x = 60$

max area = y-value of vertex.

$60 \cdot 60 = \boxed{3600}$

Since $P = 240$ and $\boxed{x = 60,\ y = 60}$

18. A hockey team plays in an arena with a seating capacity of 10,500 spectators. With the ticket price set at \$10, average attendance at recent games has been 9000. A market survey indicates that for each dollar the ticket price is lowered, the average attendance increases by 1000. $R(x) = x[(10-x)(1000) + 9000] = x(10000 - 1000x + 9000)$
$= \boxed{-1000x^2 + 19000x}$

a. What ticket price is so high that no one attends, and hence no revenue is generated?

$R(x) = 0. \Rightarrow 0 = -1000x^2 + 19000x$

$0 = x(-1000x + 19000) \Rightarrow x = 0, \boxed{\$19}$

$-1000x + 19000 = 0$

$x = 19$

b. Find the price that maximizes revenue from ticket sales.

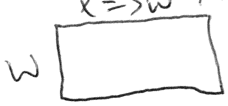
vertex!

$x = \frac{-b}{2a}$

$= \frac{+19000}{2000} = \boxed{\$9.50}$

19. A rectangular building lot is three times as long as it is wide. Find a function that models its area A in terms of its width *w*.

$x = 3w$ $A =$ length $\times$ width.



$A(w) = w \cdot 3w = \boxed{3w^2}$

20. Find a function that models the radius *r* of a circle in terms of its area *A*.

$A = \pi r^2$

(solve for "r")

$\boxed{r(A) = \sqrt{\frac{A}{\pi}}}$

21. Find a function that models the area *A* of a circle in terms of its circumference *C*.

$C = 2\pi r \Rightarrow r = \frac{C}{2\pi}$

$A = \pi r^2$

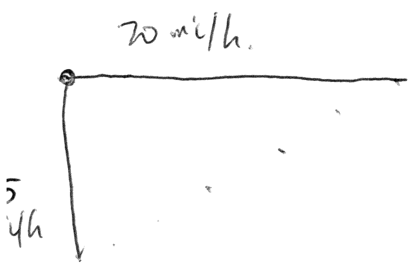
Instead of "r" we need "C"

$\Rightarrow$ $A(C) = \pi\left(\frac{C}{2\pi}\right)^2$

$= \pi\left(\frac{C^2}{4\pi^2}\right)$

$= \boxed{\frac{C^2}{4}}$

22. Two ships leave port at the same time. One sails south at 15mi/h and the other sails east at 20mi/h. Find a function that models the distance *D* between the ships in terms of the time *t* (in hours) elapsed since their departure.



Pythagorean Theorem: $a^2 + b^2 = c^2$.

$(20t)^2 + (15t)^2 = c^2$

$400t^2 + 225t^2 = c^2$

$\sqrt{625t^2} = \sqrt{c^2}$

$25t = c$

$\boxed{D(t) = 25t}$