Lesson 6-1

Slope Fields
And
Euler's Method

Objective

Students will…

- Be able to write general solution of a differential equation.
- Be able to sketch slope fields.
- Be able to approximate a solution using Euler's Method.

General Vs Particular Solution

Recall that a general form of the solution involves variables, giving you with an option to come up with any particular solution. For example, the area formula might be a general solution, which then can be used to find the area of a particular polygon with specific dimensions.

Examples

Determine whether the function is a solution of the differential equation $y'' - y = 0$.

a. $y = \sin x$

b. $y = 4e^{-x}$

c. $y = Ce^{x}$

Slope Fields

A slope field is a graph that shows the slopes at different points.
Recall that the derivative function gives the slopes of a function.

Ex. $y = 3x^2$

Examples

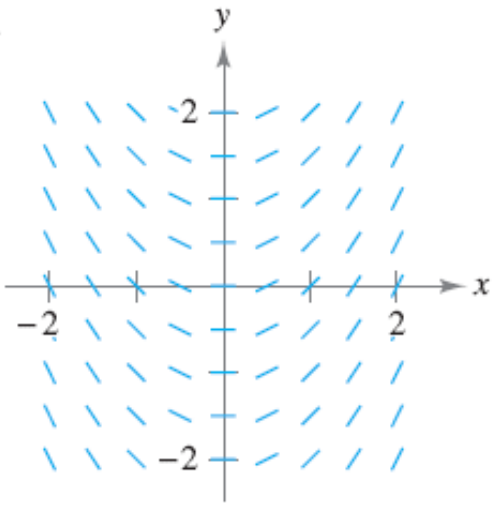
Sketch a slope field for the differential equation $y' = x - y$ for the points $(-1, 1)$, $(0, 1)$, $(1, 1)$.

Examples

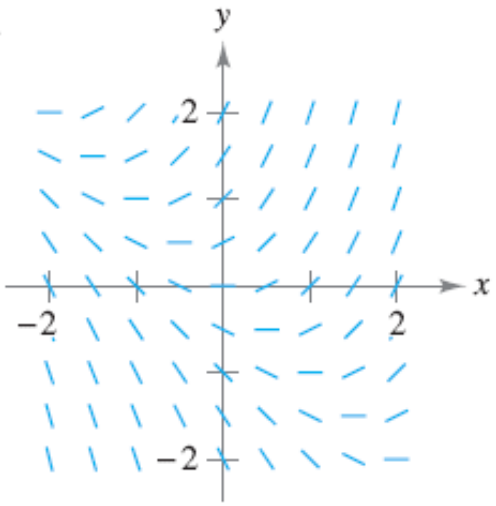
EXAMPLE 4 Identifying Slope Fields for Differential Equations

Match each slope field with its differential equation.

a.



b.



c.

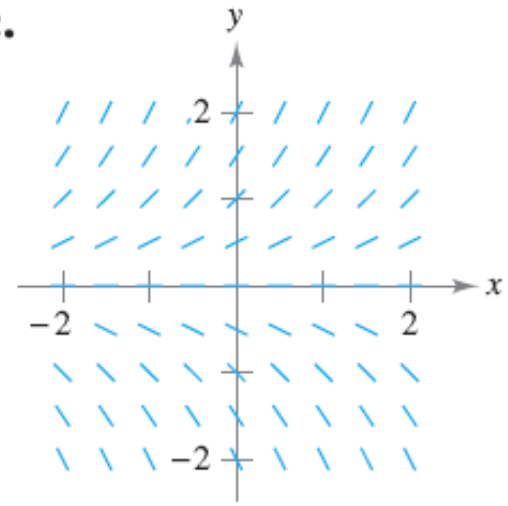


Figure 6.3

i. $y' = x + y$

ii. $y' = x$

iii. $y' = y$

Euler's Method

Euler's Method is a numerical approach to approximating the particular solution of the differential equation $y' = F(x, y)$ that passes through the point (x_0, y_0). Given a small step, say h, we can move along the tangent line until we arrive at a certain point (x_1, y_1). Moreover,

$$x_1 = x_0 + h \qquad \text{and} \qquad y_1 = y_0 + hF(x_0, y_0)$$
$$x_2 = x_1 + h \qquad\qquad y_2 = y_1 + hF(x_1, y_1)$$
$$x_3 = x_2 + h \qquad\qquad y_3 = y_2 + hF(x_2, y_2)$$
$$\vdots \qquad\qquad \vdots$$
$$x_n = x_{n-1} + h \qquad\qquad y_n = y_{n-1} + hF(x_{n-1}, y_{n-1})$$

Example

Use Euler's Method to approximate the particular solution of the differential equation: $y' = x - y$, passing through $(0, 1)$. Use a step of $h = 0.1$.

Homework 2/14

6.1 #13-23 (odd), 31-35 (odd), 53-56, 69