Lesson 2-2

Limits

Objective

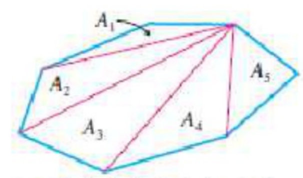
Students will…

- Be able to define limits (right vs left-hand)
- Be able to estimate limits from numerical tables.
- Be able to estimate limits from a graph.

Limits

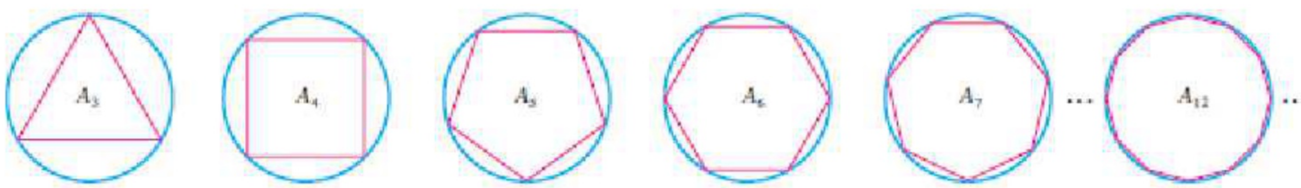
The concept of **limit** is the central idea underlying Calculus. Calculus involves focuses on studying and solving problems pertaining to situations involving **change** or **motion**. To gain a better understanding of limits consider the following example.

Finding the area of a polygon:



$A = A_1 + A_2 + A_3 + A_4 + A_5$

Finding the area of a circle:



$$\text{area} = \lim_{n\to\infty} A_n$$

Definition of Limit

Let's start by defining limit mathematically.

<u>Definition of the Limit of a Function</u>:

We write, $\lim_{x \to a} f(x) = L$, and say, "the limit of $f(x)$, as x approaches a, equals L," if we can make the values of $f(x)$ arbitrarily close to L (as close to L as we like) by taking x to be sufficiently close to a, but not equal to a.

In other words, this says that the values of $f(x)$ get closer and closer to the number L as x gets closer and closer to the number a (from either side of a) but $x \neq a$.

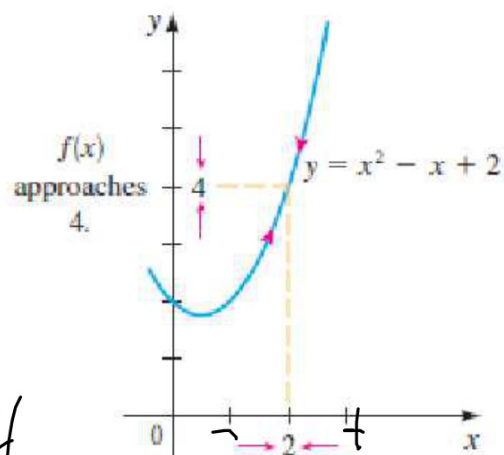
Alternative notation for $\lim_{x \to a} f(x) = L$ is $f(x) \to L$ as $x \to a$

Example $f(2) = 2^2 - 2 + 2 = 4$

Consider the following function: $f(x) = x^2 - x + 2$. Here are the tables and graph concerning this function, with x surrounding 2, but not equal to 2.

x	$f(x)$
1.0	2.000000
1.5	2.750000
1.8	3.440000
1.9	3.710000
1.95	3.852500
1.99	3.970100
1.995	3.985025
1.999	3.997001

x	$f(x)$
3.0	8.000000
2.5	5.750000
2.2	4.640000
2.1	4.310000
2.05	4.152500
2.01	4.030100
2.005	4.015025
2.001	4.003001



As x approaches 2.

$$\lim_{x \to 2} f(x) = 4$$

L

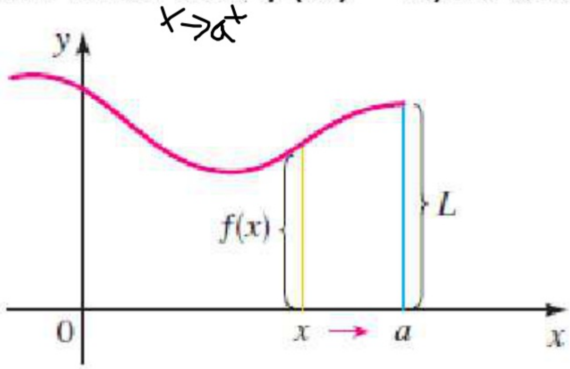
a

One-Sided Limit

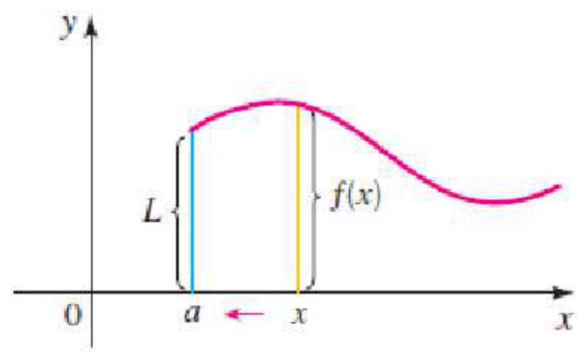
One of the ways we can consider whether a function has a limit or not, is to consider them **one side at a time**.

We write $\lim_{x \to a^-} f(x) = L$, for x approaching a from the left side.

We write $\lim_{x \to a^+} f(x) = L$, for x approaching a from the right side.



(a) $\lim_{x \to a^-} f(x) = L$



(b) $\lim_{x \to a^+} f(x) = L$

Estimating Limits

Guess the value of $\lim_{x\to 1} \boxed{\frac{x-1}{x^2-1}}$ using the table.

$= 0.5$

Guess the value of $\lim_{x\to 0} \frac{\tan 3x}{x}$

$= 3$

Existence of Limit

So in comparing the two sides of the limit, we see that the following is true.

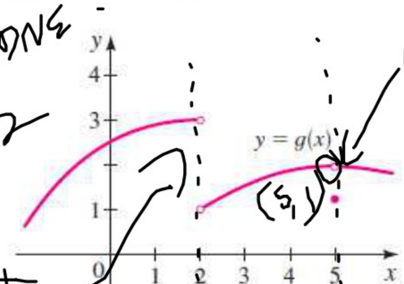
$$\lim_{x\to a} f(x) = L \qquad \text{if and only if} \qquad \lim_{x\to a^-} f(x) = L \qquad \text{and} \qquad \lim_{x\to a^+} f(x) = L$$

Ex. The graph of a function g is shown in Figure 10. Use it to state the values (if they exist) of the following:

(a) $\lim_{x\to 2^-} g(x)$, 3 $\quad \lim_{x\to 2^+} g(x)$, 1 $\quad \lim_{x\to 2} g(x)$ = DNE

(b) $\lim_{x\to 5^-} g(x)$, $\quad \lim_{x\to 5^+} g(x)$, $\quad \lim_{x\to 5} g(x)$ 2

2 $\qquad$ 2



Removable Discont.

(5, 1)

$g(5) = 1$

Jump Disc.

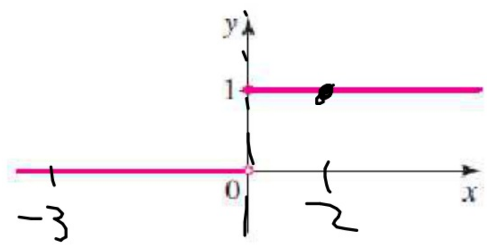
Limits that Fail to Exist

There are certain occasions where the limit does not exist.

1. A function with a "Jump."

Ex.

$$H(t) = \begin{cases} 0 & \text{if } t < 0 \\ 1 & \text{if } t \geq 0 \end{cases}$$



Limit as $t \to 0$ does not exist.

2. A function that oscillates.

Ex.

$$\lim_{x \to 0} \sin \frac{\pi}{x}.$$

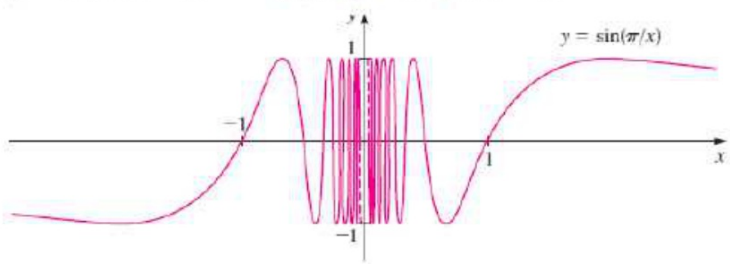
$f(1) = \sin \pi = 0$ $\quad$ $f(\frac{1}{2}) = \sin 2\pi = 0$

$f(\frac{1}{3}) = \sin 3\pi = 0$ $\quad$ $f(\frac{1}{4}) = \sin 4\pi = 0$

$f(0.1) = \sin 10\pi = 0$ $\quad$ $f(0.01) = \sin 100\pi = 0$

Is it zero? However…

Limit does not exist!



Homework 8/30

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