Lesson 1-4

Continuity

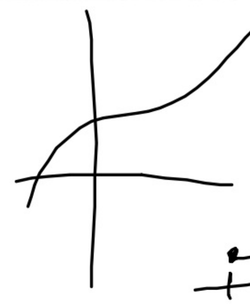
Objective

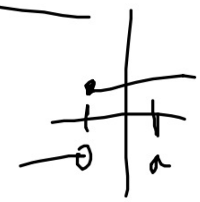
Students will…

- Be able to distinguish between removable and nonremovable discontinuities.
- Be able to define and use the intermediate value theorem.

Continuity

A function, say f, is **continuous at c** when these three conditions are met:





1. $f(c)$ is defined. (i.e. can be evaluated).
2. $\lim_{x \to c} f(x)$ exists.
3. $\lim_{x \to c} f(x) = f(c)$.

Recall: We can show that a function has a limit at any given point by the existence of limit theorem:

$\lim_{x \to c^-} f(x) = L = \lim_{x \to c^+} f(x)$, (the right and the left side limits are equal)

$x-1$
x^2-1

Types of Discontinuity

Always remember that not all discontinuities are created equal! In fact, just because a discontinuity exists at a certain point, this doesn't automatically indicate that the limit doesn't exist. Consider the following problems:

a. $\lim_{x\to1}\left(\frac{x^2-1}{x-1}\right) = \frac{(x+1)\cancel{(x-1)}}{\cancel{x-1}} = (x+1)$

b. $\lim_{x\to1}\frac{1}{x-1} = \text{DNE.} = \infty.$

$\lim_{x\to1}(x+1) = \boxed{2}$

Types of Discontinuity

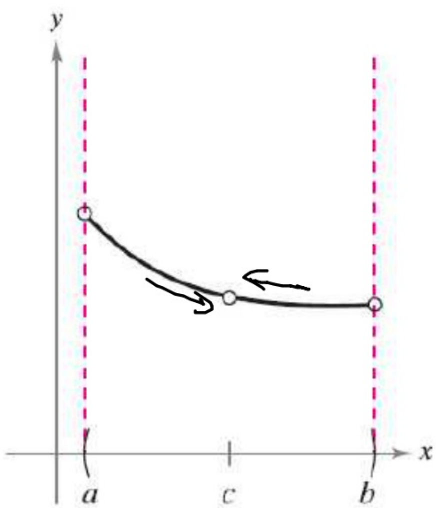
Clearly, (a) has a limit, while (b) did not. Algebraically speaking, simple factoring and simplifying allowed us to find the limit for (a), while there was nothing that could have been done for to find a limit for (b). This can be more easily seen looking at their graphs.

In general…

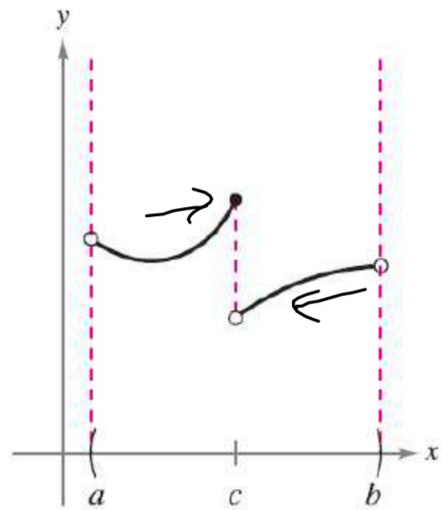
If the limit exists at a certain point of a function, say c, while the function is undefined at c, then the function is said to have a **removable discontinuity** at c

If the limit does not exist at c, nor is defined at c, then the function is said to have a **nonremovable discontinuity** at c.

Removable vs Nonremovable Discontinuity



(a) Removable discontinuity



(b) Nonremovable discontinuity

Examples

Find points of discontinuity, and determine if they are removable, or nonremovable discontinuity (ies).

a. $f(x) = \frac{4}{x-6}$

$x \neq 6$ nonremov.

b. $f(x) = \frac{x-5}{x^2-25} = \frac{\cancel{x-5}}{(x+5)\cancel{(x-5)}}$

$x^2-25=0$

$x=\pm 5$

-5 nonremov.

5 remov.

c. $f(x) = \frac{\cancel{x+2}}{x^2-x-6}$

-2 remov.

3 nonremov.

$x^2-x-6=0$

$\cancel{(x+2)}(x-3)=0$ $\quad x+2=0$ $\quad x-3=0$

$-2 \quad\quad 3$

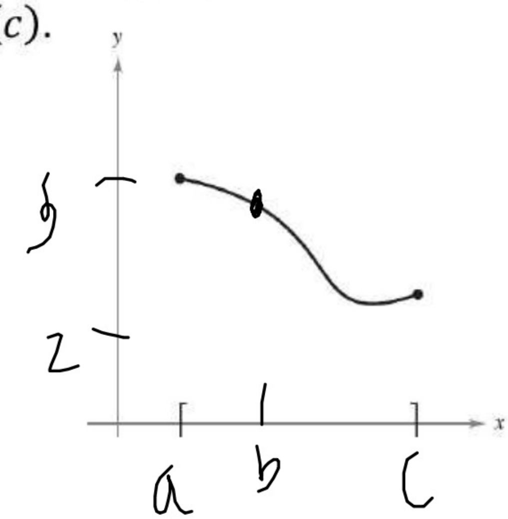
$x=-2, 3$

Intermediate Value Theorem

There is a very simple but important theorem in Calculus regarding continuity.

<u>Intermediate Value Theorem</u>- If f is continuous on the closed interval $[a, c]$, and $f(a) \neq f(c)$, and k is any number between $f(a)$ and $f(c)$, then there is at least one number b in $[a, c]$ such that $f(b) = k$.

In other words, in the interval $[a, \overset{c}{\cancel{b}}]$, if $a \leq b \leq c$, then, $f(b)$ exists, such that $f(a) \leq f(b) \leq f(c)$.

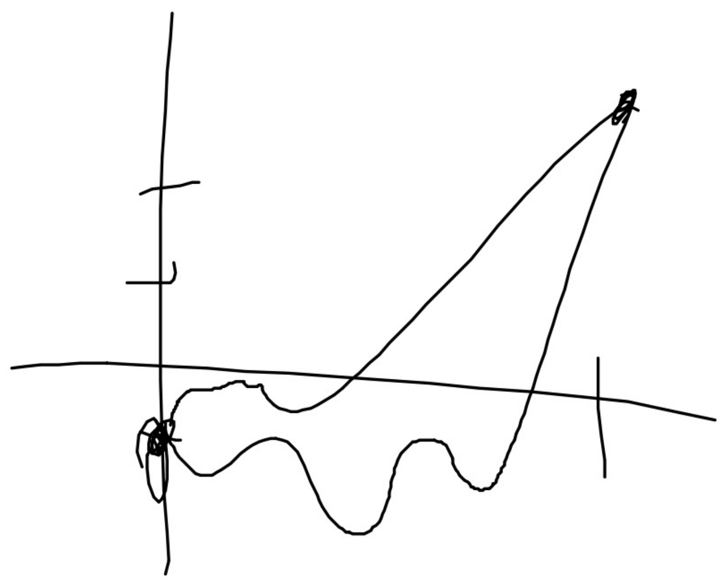


Example

Use the Intermediate Value Theorem to show that the polynomial function $f(x) = x^3 + 2x - 1$ has a zero (x-intercept or root) in the interval $[0,1]$.

$f(0) = -1$

$f(1) = 2$



Example

Use the Intermediate Value Theorem to show that the function

$f(x) = -\frac{5}{x} + \tan\frac{\pi x}{10}$ has a zero (x-intercept or root) in the interval $[1,4]$.

Homework 9/11

~~1.4 exercises #35-47 (odd), 48, 51, 53, 83-84~~

1.4 ex. 7-23 (e.o.o), 25-28, 31, 33-53 (e.o.o), 83, 86