Lesson 10-3b

Parametric Equation II

Objective

Students will…

- Be able to find the arc length of parametric equations.
- Be able to convert from rectangular coordinates to polar and vice-versa.

Arc Length Parametric Equation

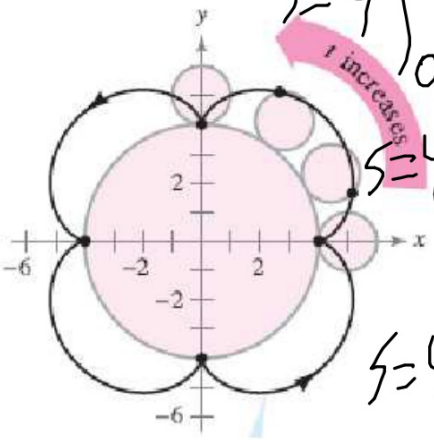
The arc length formula for parametric equations is similar to the one for rectangular equations.

Arc Length in Parametric Form- $s = \int_a^b \sqrt{[f'(t)]^2 + [g'(t)]^2}\, dt$

$$S = \int_a^b \sqrt{(x')^2 + (y')^2}$$

A circle of radius 1 rolls around the circumference of a larger circle of radius 4. The epicycloid traced by a point on the circumference of the smaller circle is given by $x = 5\cos t - \cos 5t$ and $y = 5\sin t - \sin 5t$.

Find the distance traveled by the point in one complete trip about the larger circle.



$$S = 4\int_0^{\pi/2} \sqrt{(-5\sin t + 5\sin 5t)^2 + (5\cos t - 5\cos 5t)^2}\, dt$$

$$S = 4\int_0^{\pi/2} \sqrt{25\sin^2 t - 50\sin t\sin 5t + 25\sin^2 5t + 25\cos^2 t - 50\cos t\cos 5t + 25\cos^2 5t}\, dx$$

$$S = 4\int_0^{\pi/2} \sqrt{25 + 25 - 50\sin t\sin 5t - 50\cos t\cos 5t}\, dx$$

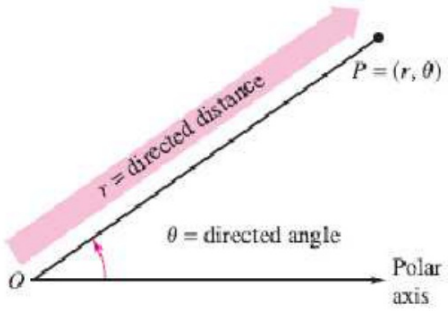
$$S = 20\int_0^{\pi/2} \sqrt{2 - 2\sin t\sin 5t - 2\cos t\cos 5t} = 40\int_0^{\pi/2} \sqrt{2 - 2\cos 4t}$$

$$= 40$$

Polar Coordinates

(x, y) – Rectangular

You may recall polar coordinates. They are in the form (r, θ). r represents the directed distance from the origin, and θ is the directed angle going counter-clockwise.



Polar coordinates
Figure 10.36

Conversion of the Coordinates

The polar coordinates (r, θ) of a point are related to the rectangular coordinates (x, y) of the point as follows.

1. $x = r\cos\theta, y = r\sin\theta$ (Polar → rect.)
2. $\tan\theta = \frac{y}{x}, r^2 = x^2 + y^2$ (rect → polar)

Examples

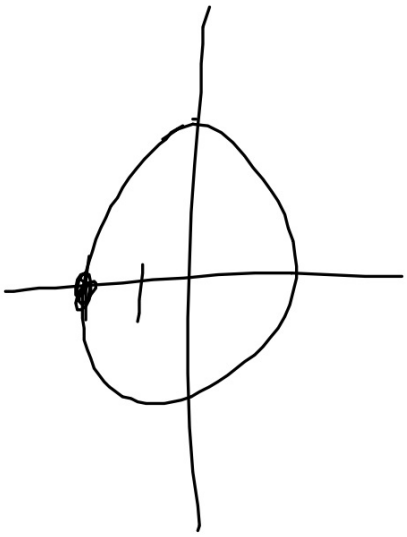
Convert polar coordinate $(2, \pi)$ in to rectangular coordinates.

r, θ $\qquad$ (x, y) ?

$x = r\cos\theta = 2\cos\pi = 2(-1) = -2$

$y = r\sin\theta = 2\sin\pi = 2(0) = 0$

$(-2, 0)$



Examples

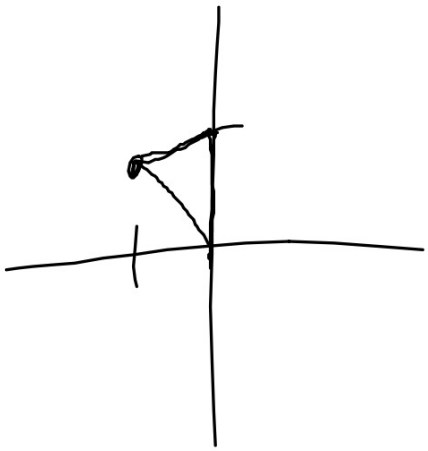
(x, y) (r, θ)

Convert rectangular coordinate $(-1, 1)$ in to polar coordinates.

$r^2 = x^2 + y^2 \Rightarrow r^2 = (-1)^2 + (1)^2 = 1 + 1 = 2 \Rightarrow r = \pm\sqrt{2}$

$\tan\theta = \frac{y}{x} = \frac{1}{-1} = -1 \Rightarrow \theta = 3\pi/4$

$(\sqrt{2}, 3\pi/4)$



Homework 4/20

10.3 #43-46, 47, 49

10.4 #27-41 (odd)