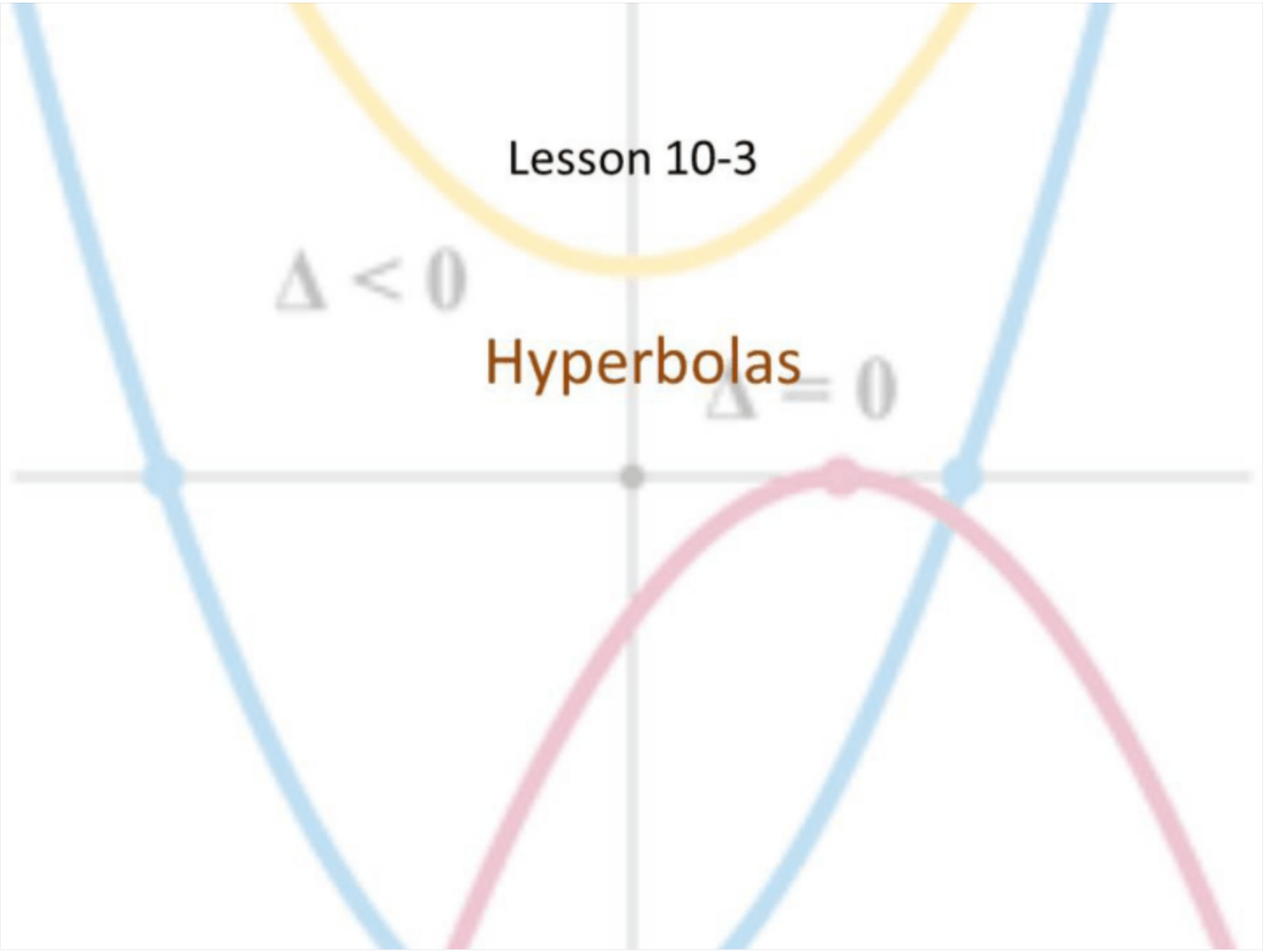

Lesson 10-3
$\Delta < 0$
Hyperbolas
$\Delta = 0$

Objective

Students will…

- Be able to know the geometric definition of a hyperbola.
- Be able to know and use the standard equation of hyperbolas and sketch their graphs.

Hyperbolas within a Cone

A hyperbola can be cut out from a cone.

Hyperbola

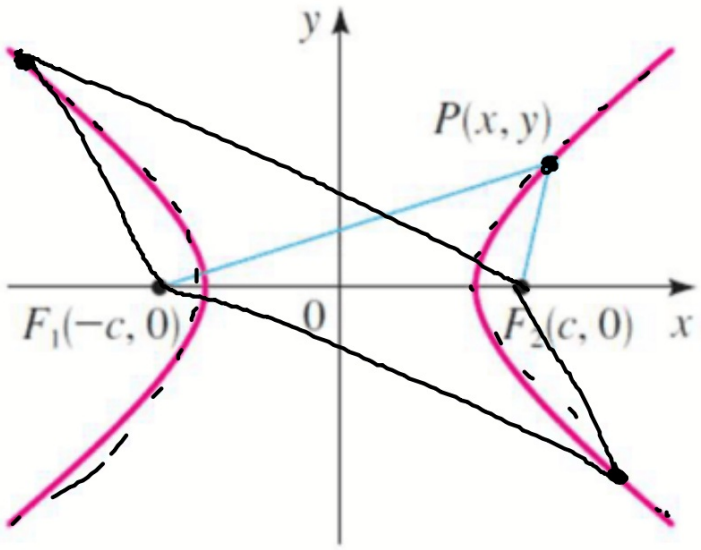
The shape of a cooling tower is a hyperbola.

Hyperbola

Here, we want to geometrically define what a hyperbola is.

Geometric Definition of a hyperbola- Is the set of all points in the plane, the difference of whose distances from two fixed points F_1 and F_2 is a constant. These two fixed points are the **foci** of the hyperbola.

Ex.

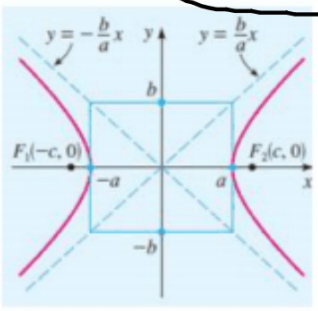


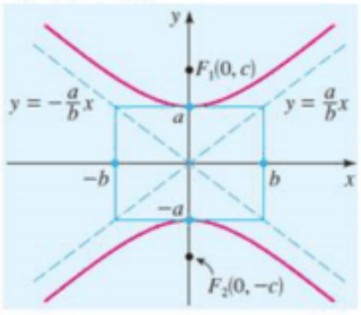
Equations and Graphs of Hyperbolas

Using the distance formula, we can see that parabolas have the following equations: ***for $a > 0$*** and ***$b > 0$*** (**not** $a > b$)

Horizontal	**Vertical**
$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$	$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$
Vertices: $(\pm a, 0)$	$(0, \pm a)$
Covertices: $(0, \pm b)$	$(\pm b, 0)$
Transverse Axis: Horizontal length 2a	Vertical length $2a$
Asymptotes: $y = \pm \frac{b}{a} x$	$y = \pm \frac{a}{b} x$
Foci: $(\pm c, 0)$, $c^2 = a^2 + b^2$	$(0, \pm c)$, $c^2 = a^2 + b^2$

$\frac{-y^2}{b^2} + \frac{x^2}{a^2} = 1$



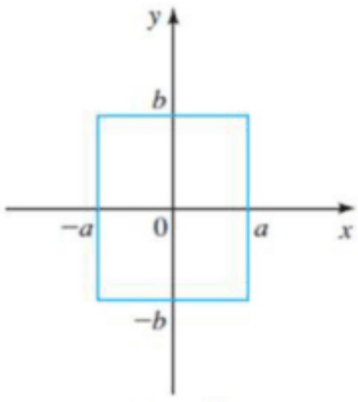


Sketching the Hyperbola

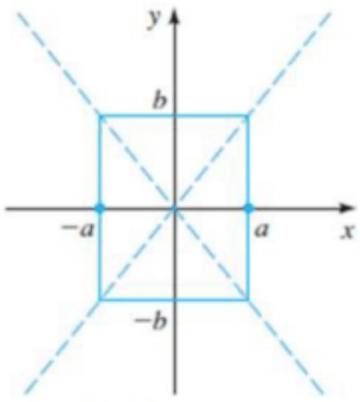
Here is a guidelines you can use to graph the hyperbola.

1. Sketch the **central box**, using the vertices and the covertices.
2. Sketch the **asymptotes**. These are the diagonals of the central box.
3. Plot the foci
4. Sketch the hyperbola.

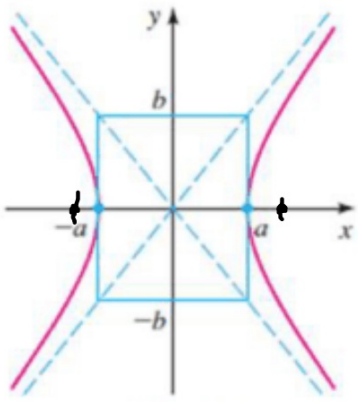
Ex.



(a) Central box



(b) Asymptotes



(c) Hyperbola

Example

Find the vertices, covertices, foci, and the asymptotes. Sketch the graph.

$$\frac{9x^2}{144} - \frac{16y^2}{144} = \frac{144}{144} \Rightarrow \frac{x^2}{16} - \frac{y^2}{9} = 1$$

Horiz.

Vert: $(\pm a, 0) = (\pm 4, 0)$

Covert: $(0, \pm b) = (0, \pm 3)$

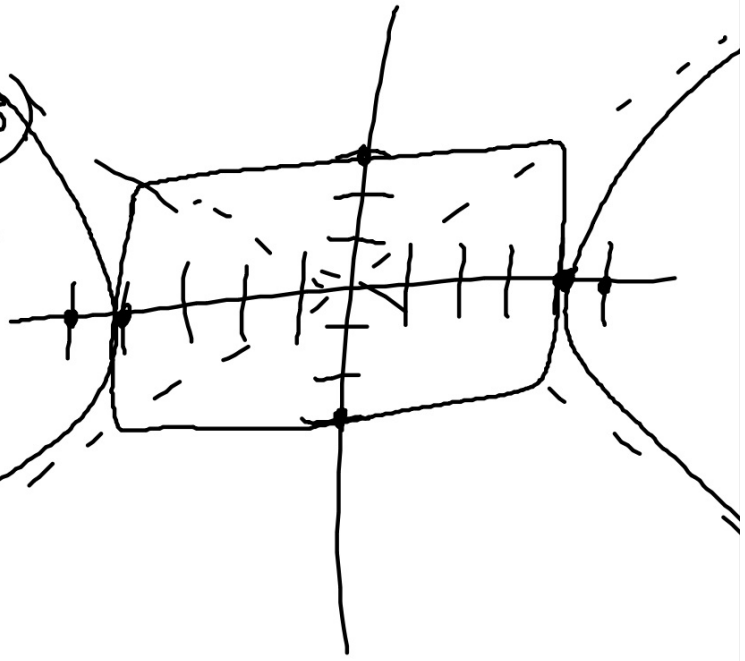
Foci: $(\pm c, 0) = (\pm 5, 0)$

$c^2 = a^2 + b^2$

$c^2 = 16 + 9$

$c = \pm 5$

asym: $y = \pm \frac{3}{4}x$



Example

Find the vertices, covertices, foci, and the asymptotes. Sketch the graph.

$25x^2 - 144y^2 = 3600$

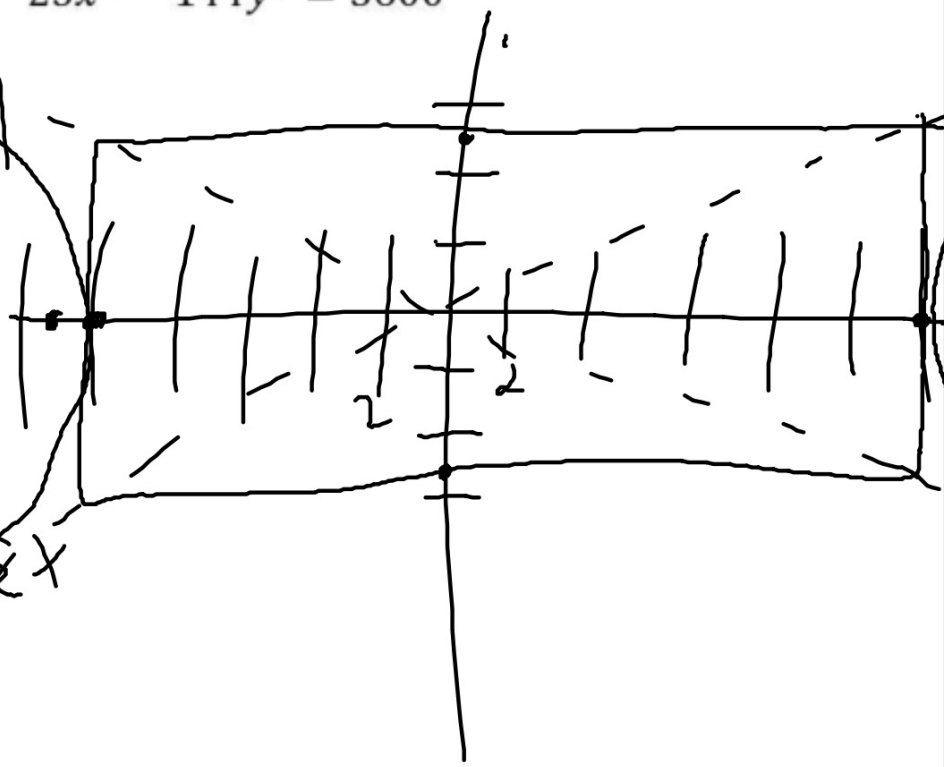
$\frac{x^2}{144} - \frac{y^2}{25} = 1$

Vert.: $(\pm 12, 0)$

covert: $(0, \pm 5)$

foci: $(\pm 13, 0)$

asym: $y = \pm \frac{5}{12}x$



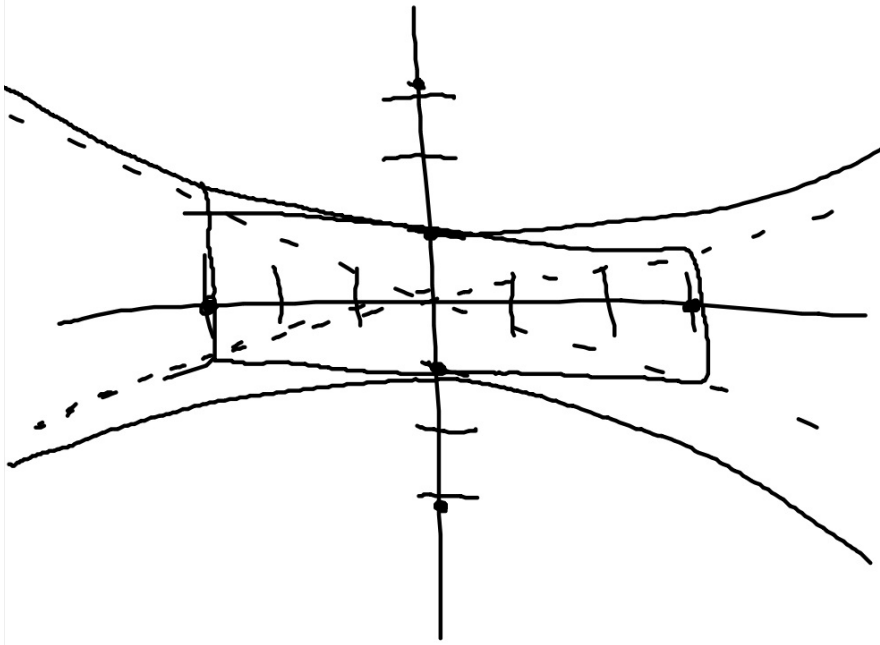
Example

Find the vertices, covertices, foci, and the asymptotes. Sketch the graph.

$x^2 - 9y^2 + 9 = 0$

$-9 \quad -9$

$\Rightarrow \frac{x^2}{-9} - \frac{9y^2}{-9} = \frac{-9}{-9}$



$\Rightarrow -\frac{x^2}{9} + \frac{y^2}{1} = 1$

$\Rightarrow \frac{y^2}{1} - \frac{x^2}{9} = 1$

Vert: $(0, \pm 1)$

Covert: $(\pm 3, 0)$

foci: $(0, \pm\sqrt{10})$

asym: $y = \pm \frac{1}{3}x$

Example

Find the vertices, covertices, foci, and the asymptotes. Sketch the graph.

$$\frac{y^2}{4} - x^2 = 1$$

Homework 4/10

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