Lesson 10-2, 10-3a

Parametric Equation

Objective

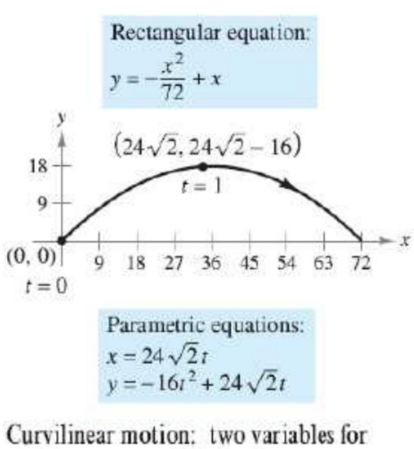
Students will…

- Be able to define parametric equation.
- Be able to sketch the graph of a curve given by a set of parametric equations.
- Be able to find the slope and the tangent line of parametric equations.

Parametric Equation

Until now, you have been representing a graph by a single equation involving two variables. But, Calculus sometimes requires for a third variable, time. This is what we call a parametric equation.

<u>Parametric Equation</u>- If f and g are continuous functions of t on an interval I, then the equations $x = f(t)$ and $y = g(t)$ are called parametric equations and t is called a parameter.

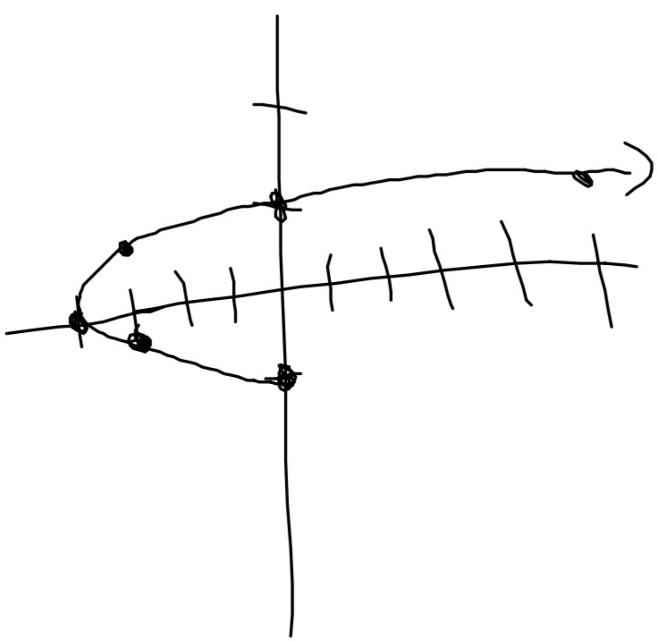


Curvilinear motion: two variables for position, one variable for time
Figure 10.19

Sketching Parametric Equations

Example: Sketch the curve described by the parametric equations

$x = t^2 - 4$ and $y = \frac{t}{2}$, $-2 \leq t \leq 3$



t	$(x$	$y)$
-2	0	-1
-1	-3	$-1/2$
0	-4	0
1	-3	$1/2$
2	0	1
3	5	$3/2$

Finding Equations for a Given Graph

Find a set of parametric equations to represent the graph of $y = 1 - x^2$, using each of the following parameters: $t = x$ and the slope $m = \frac{dy}{dx}$ at the point (x, y).

$x = t$

$\frac{dy}{dx} = -2x = m$

$m = -2x = -2t$

$x = -2t$

$y = 1 - (-2t)^2$

$= \boxed{1 - 4t^2}$

Derivative of Parametric Equations

If a smooth curve C is given by the equations $x = f(t)$ and $y = g(t)$, then the slope of C at (x, y) is

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

$$\frac{d^2y}{(dx)^2} = \frac{\left(\frac{dy}{dx}\right)'}{dx/dt}$$

In other words, take the derivative of the y equation and put it over the derivative of the x equation.

Examples

Find $\frac{dy}{dx}$ for the curve given by $x = \sin t$ and $y = \cos t$.

$$\frac{dy}{dt} = -\sin t$$

$$\frac{dx}{dt} = \cos t$$

$$\frac{dy}{dx} = \frac{-\sin t}{\cos t} = \boxed{-\tan t}$$

$$\frac{d^2y}{(dx)^2} = \frac{(dy/dx)'}{dx/dt} = \frac{(-\tan t)'}{\cos t} = \frac{-\sec^2 t}{\cos t} = \boxed{-\sec^3 t}$$

$2 = \sqrt{t}$ | $3 = 1/4(t^2 - 4)$

$4 = t$ | $12 = t^2 - 4$

$16 = t^2$

$t = 4$

Examples

For the curve given by $x = \sqrt{t}$ and $y = \frac{1}{4}(t^2 - 4)$, $t \geq 0$, find the slope and concavity at the point $(2, 3)$.

$\frac{dy}{dx} =$

$\frac{dy}{dt} = \frac{1}{4}(2t)$

$= \frac{1}{2}t$

$\frac{dx}{dt} = \frac{1}{2}t^{-1/2}$

$\frac{dy}{dx} = \frac{\cancel{\frac{1}{2}}t}{\cancel{\frac{1}{2}}t^{-1/2}} = t^1 t^{1/2} = \boxed{t^{3/2}}$

$m = (4)^{3/2} = \boxed{8}$

$\frac{(t^{3/2})'}{\frac{1}{2}t^{-1/2}} = \frac{\frac{3}{\cancel{2}}t^{1/2}}{\cancel{\frac{1}{2}}t^{-1/2}} = 3t^{1/2} \cdot t^{1/2}$

$= 3t \Rightarrow 3(4) = 12$

Concave up

Homework 4/19

10.2 #1, 3, 9, 43, 44

10.3 #5-13 (odd)