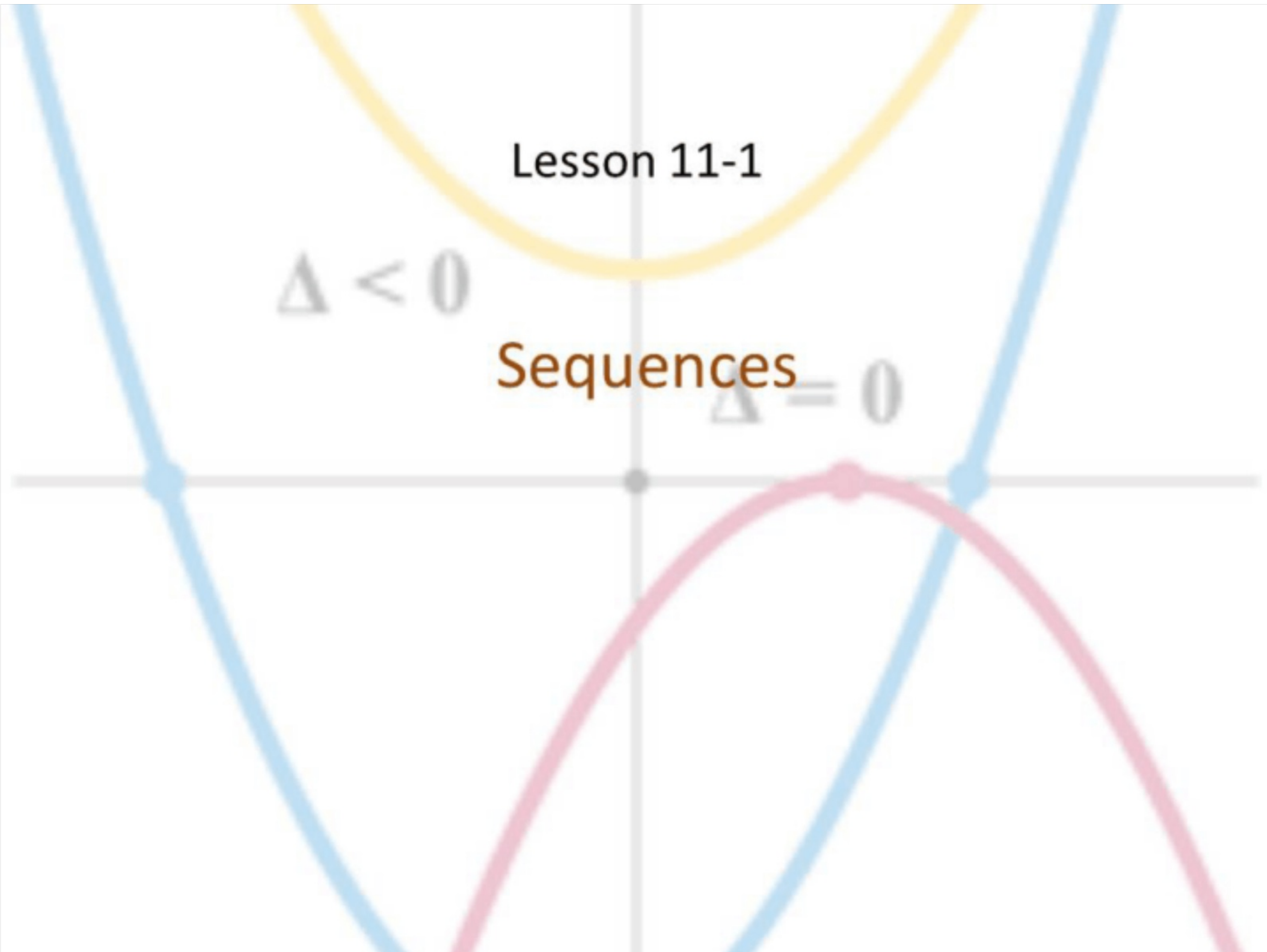
Lesson 11-1

Sequences



Objective

Students will…

- Be able to define and differentiate the different types of sequences (including Fibonacci)
- Be able to derive the sequence from its formula.
- Be able to derive the formula from the sequence.

Sequences

Many real-world processes generate list of numbers. For example, games scores, bank account numbers, etc. In mathematics, we call such **sequences**.

A sequence is a set of numbers written in a specific order ...

$$a_1, a_2, a_3, \dots, a_n$$

Here, a_1 is called the first term, a_2 is the second term, and so on. And a_n is the nth term of the sequence. Let's see how this look on a table:

n	1	2	3	4...	n
a_n	a_1	a_2	a_3	a_4 ...	a_n

Here, we can see that any given sequence can be written as a function.

Functional Definition of a Sequence

A **sequence** is a function f whose domain is the set of natural numbers. The values $f(1), f(2), f(3)$,… are called the **terms** of the sequence.

With this definition we can write a sequence using a formula.

Ex. $a_n = 2n$, where n represents the term number, i.e. a_n is the nth term of the sequence.

So get the first term, we plug in 1 for n, and so on…

$a_1 = 2(1) = 2, a_2 = 2(2) = 4, a_3 = 2(3) = 6, \ldots$

We can even represent the sequence using a table.

1	2	3	4	5…
2	4	6	8	10 …

Example

Find the first five terms and the 100th term of the sequence defined by each formula (rule).

a. $a_n = 2n - 1$

$a_1 = 2(1) - 1 = 1$

$a_2 = 2(2) - 1 = 3$

$a_3 = 2(3) - 1 = 5$

$a_4 = 2(4) - 1 = 7$

$a_5 = 2(5) - 1$

$a_{100} = 2(100) - 1$

b. $c_n = n^2 - 1$

$0, 3, 8, 15, 24, \ldots, 9999.$

c. $r_n = \frac{(-1)^n}{2^n}$

$-\frac{1}{2}, \frac{1}{4}, -\frac{1}{8}, \frac{1}{16}, -\frac{1}{32}, \ldots \frac{1}{2^{100}}.$

Finding the Formula

$x^a x^b = x^{a+b}$

As discussed, a sequence may be derived from a formula. A much more difficult task consists of deriving the formula from the sequence. This could become a rigorous process of identifying the pattern associated with each sequence. In other words, there's may not be an easy way out!

(Hint: set them as a table)

Examples:

$\frac{1}{2}, \frac{3}{4}, \frac{5}{6}, \frac{7}{8}$

$a_n = \frac{2n-1}{2n}$

ex. $2, -4, 8, -16, 32$

$-1(-1)^n 2^n = (-1)^{n+1} 2^n$

$-2, 4, -8, 16, -32$

$a_n = (-1)^n 2^n$

n	1	2	3	4	5
a_n	-2	4	-8	16	-32

Recursively Defined Sequences

Some sequences may be derived from "within." In other words, writing of each term may be dependent on its preceding term (the term before it). These sequences are called **recursive sequences**. Here is an example:

Find the first five terms of the sequence defined recursively by $a_1 = 1$ and $a_n = 3(a_{n-1} + 2)$.

$a_1 = 1$

$a_2 = 3(a_{2-1} + 2) = 3(a_1 + 2) = 3(1+2) = 9$

$a_3 = 3(a_{3-1} + 2) = 3(a_2 + 2) = 3(9+2) = 33$

$a_4 = 3(33+2) = 3(35) = 105$

$a_5 = 3(105+2) = 3(107) = 321$

Fibonacci Sequence

The most famous recursive sequence would be the **Fibonacci Sequence**.

Fibonacci Sequence: Sequence written by the formula: $F_1 = 1$, $F_2 = 1$, and $F_n = F_{n-1} + F_{n-2}$

$F_3 = F_{3-1} + F_{3-2} = F_2 + F_1 = 1 + 1 = 2$.

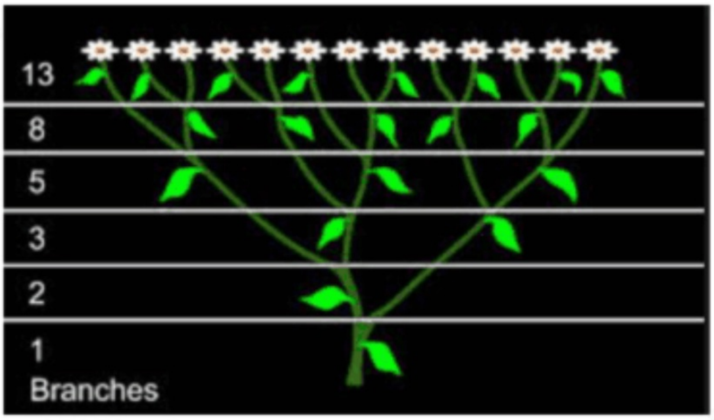
Example: Write the first 11 terms of the Fibonacci Sequence

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89.

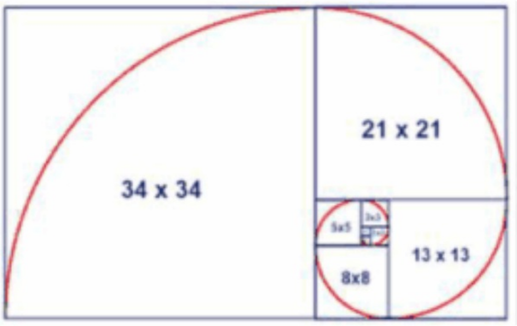
Fibonacci in Real Life

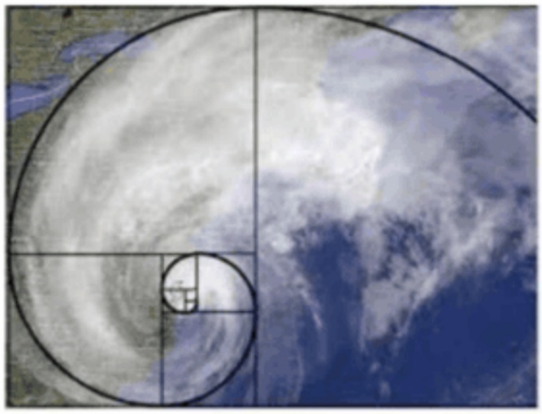
The reason why the Fibonacci Sequence is renown is because of the fact that so many natural phenomena behave like it. In fact there is a mathematical journal, *Fibonacci Quarterly* that is entirely devoted to its properties.

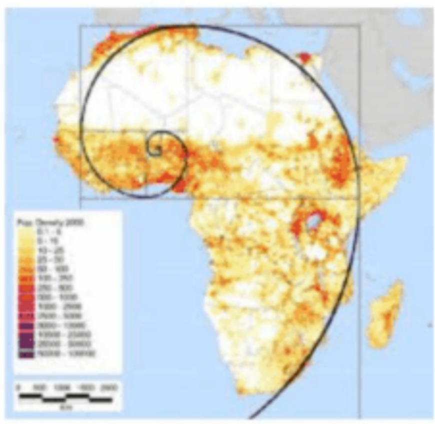
Ex.



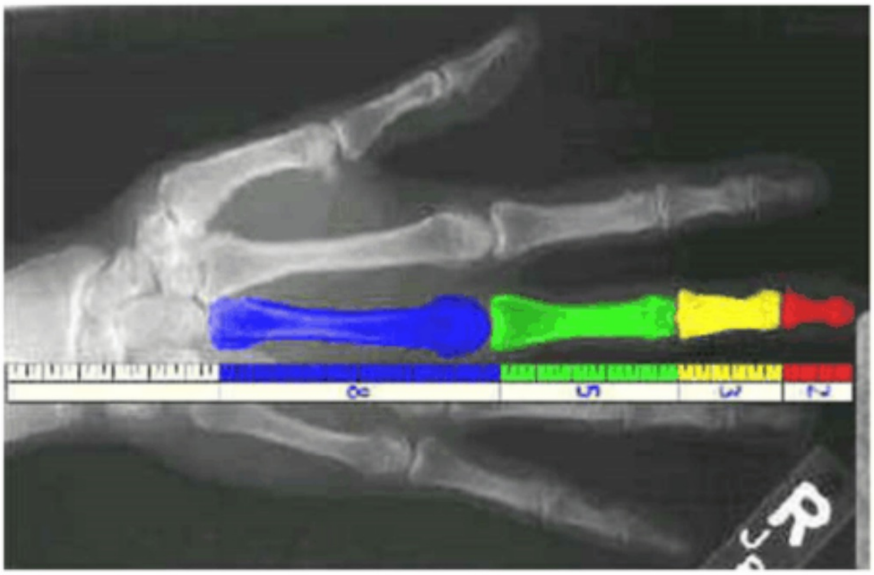
Fibonacci in Real Life







Fibonacci in Real Life



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Homework 4/16

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