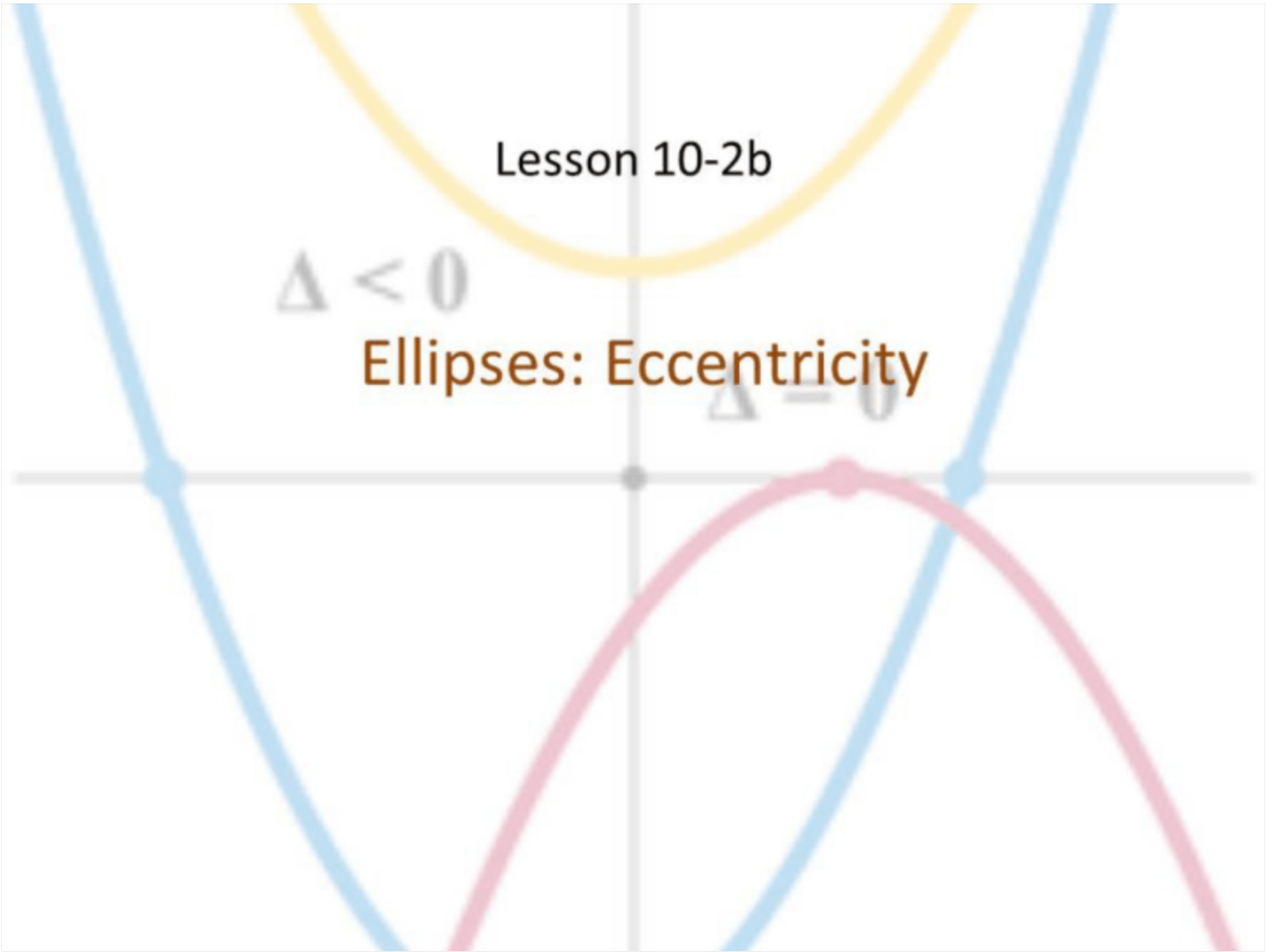


Lesson 10-2b

$\Delta < 0$

Ellipses: Eccentricity

$\Delta = 0$



## Objective

Students will...

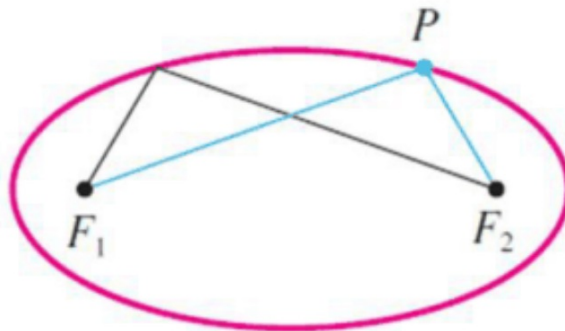
- Be able to know and set up the ratio of eccentricity for any given ellipse.
- Be able to derive equation of an ellipse given the eccentricity and the foci.

# Ellipse

Here, we want to geometrically define what an ellipse is.

**Geometric Definition of an Ellipse**- An ellipse is the set of all points in the plane the sum whose distances from two fixed points  $F_1$  and  $F_2$  is a constant. These two fixed points are **foci** (plural of focus) of the ellipse.

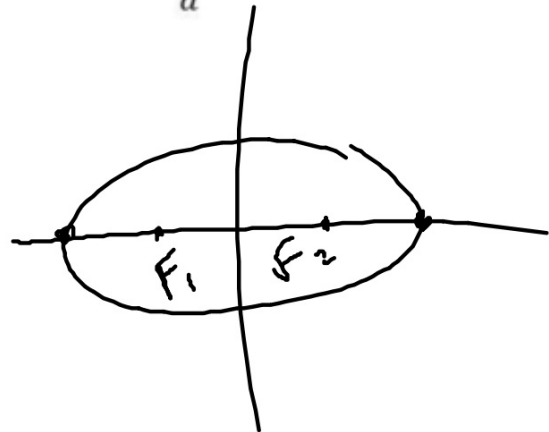
Ex.



## Eccentricity

We know that an ellipse is an oval shape, which can be considered as a **stretched circle**. A numerical value that indicates how close an ellipse is to a circular shape is known as the **eccentricity** of a circle.

**Eccentricity**- Given a standard ellipse with the equation  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  or  $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$ , the eccentricity  $e$  is the number  $e = \frac{c}{a}$ , where  $c^2 = a^2 - b^2$  and for every ellipse,  $0 < e < 1$ .



## Eccentricity of an Ellipse

With that being said, eccentricity is useful in telling us just how close an ellipse looks like a circle. If  $e$  is closer to 0, then the ellipse looks more like a circle. If  $e$  is closer to 1, then the ellipse looks more like an oval.



$e = 0.1$



$e = 0.5$



$e = 0.68$



$e = 0.86$

## Example

Find the eccentricity of the following ellipse.

$$y = \frac{x^2}{25} + \frac{y^2}{9} = 1$$

$$e = \frac{c}{a} = \frac{4}{5} = \textcircled{0.8}$$

$$c^2 = a^2 - b^2$$

$$c^2 = 25 - 9$$

$$c^2 = 16$$

$$c = \pm 4$$

## Example

Find the eccentricity of the following ellipse.

$$y = \frac{x^2}{16} + \frac{y^2}{25} = 1$$

$$e = \frac{c}{a} = \frac{3}{5} = 0.6$$

$$c^2 = 25 - 16$$
$$= 9$$

$$c = \pm 3$$

### Example

Find the equation of the ellipse with foci  $(0, \pm 8)$  and eccentricity  $e = \frac{4}{5}$ .

$$\frac{y^2}{a^2} + \frac{x^2}{b^2} = 1$$

$$c^2 = a^2 - b^2$$

$$64 = 100 - b^2$$

$$b^2 = 36$$

$$\frac{y^2}{100} + \frac{x^2}{36} = 1$$

$$e = \frac{c}{a} = \frac{4}{5}$$

$\Rightarrow \frac{8}{a} = \frac{4}{5}$   
 $\Rightarrow a = 10$



### Example

Find the equation of the ellipse with foci  $(0, \pm 20)$  and eccentricity  $e = \frac{4}{5}$ .

$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$$

$$= \frac{20}{25} = a$$

$$c^2 = a^2 - b^2$$
$$400 = 625 - b^2$$
$$b^2 = 225$$

$$\frac{x^2}{225} + \frac{y^2}{625} = 1$$

## Homework 5/21

TB pg. 759-760 #9-15 (e.o.o) (just find the  
eccentricity!), 38-40