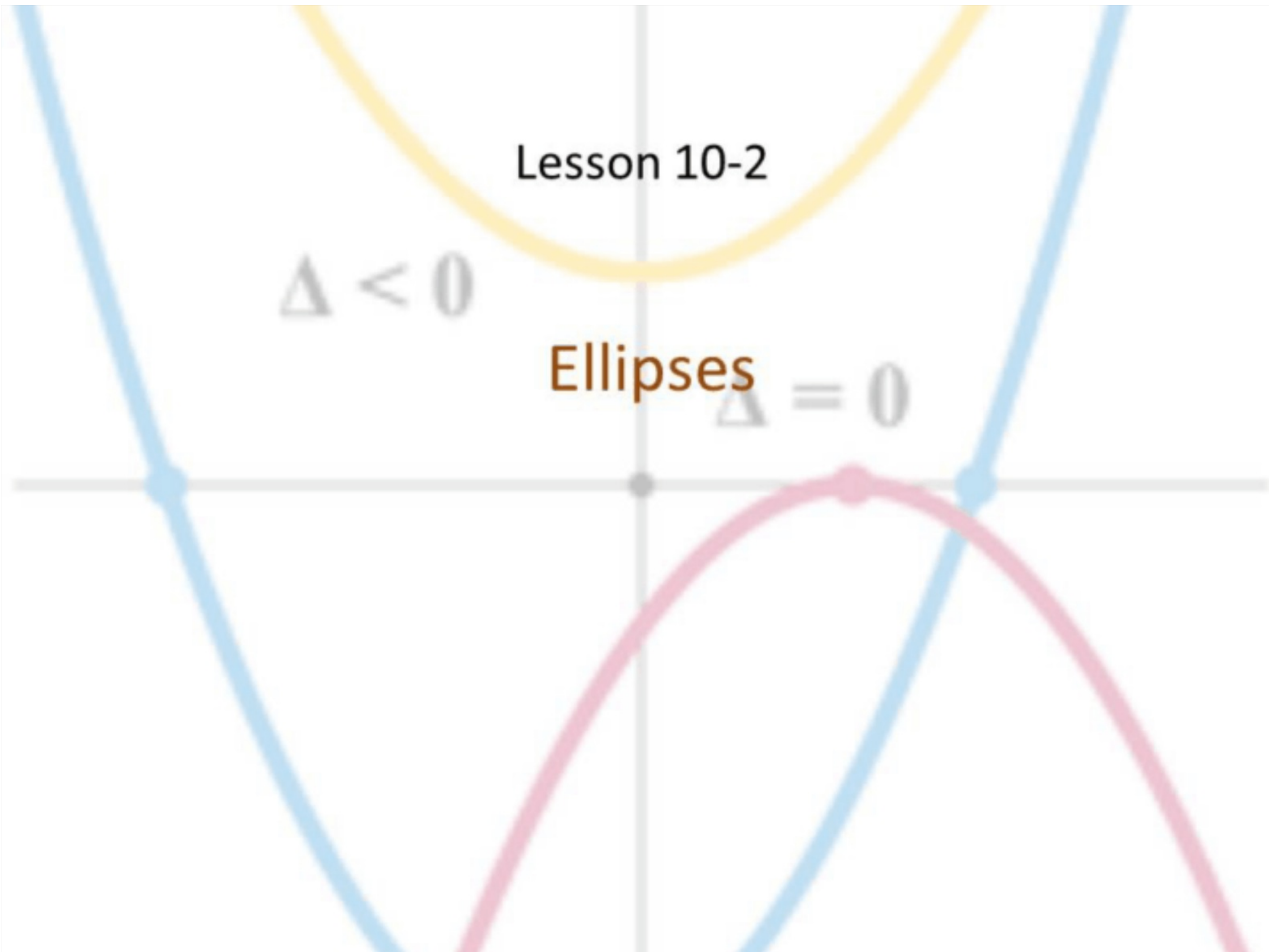
Lesson 10-2

Ellipses



Objective

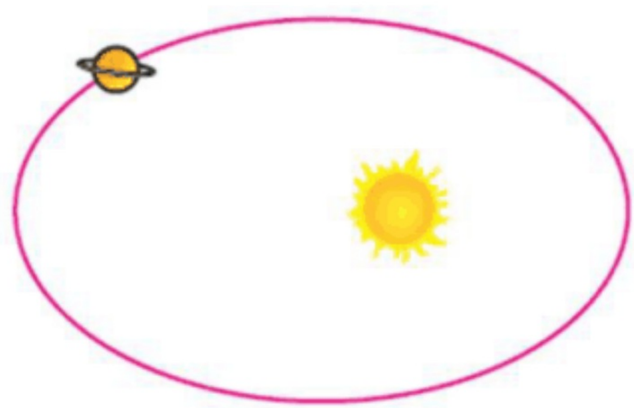
Students will…

- Be able to give a geometric definition of an ellipse.
- Be able to know the standard equation of ellipses.

Ellipse within a Cone

As seen from yesterday's video, a parabola can be cut out from a cone. Parabolas are easily found in the real-world.

Ellipse

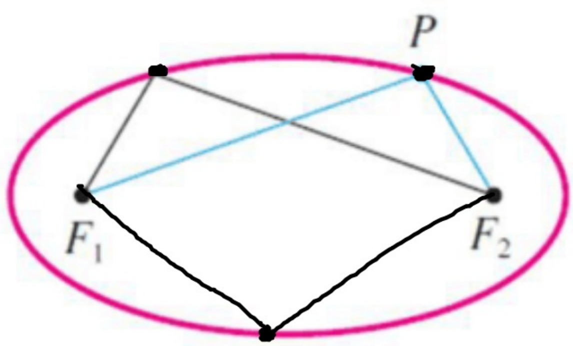


Ellipse

Here, we want to geometrically define what an ellipse is.

<u>Geometric Definition of an Ellipse</u>- An ellipse is the set of all points in the plane the sum whose distances from two fixed points F_1 and F_2 is a constant. These two fixed points are **foci** (plural of focus) of the ellipse.

Ex.



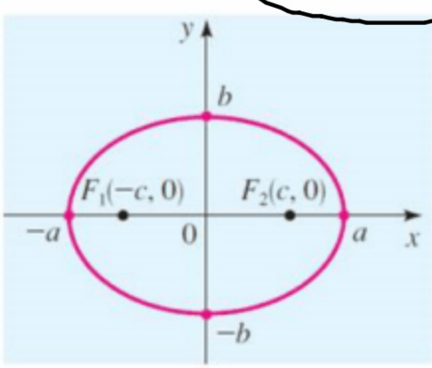
$\frac{x^2}{9} + \frac{y^2}{10} = 1$

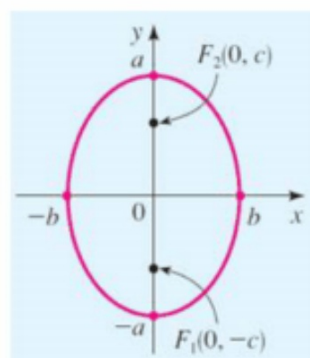
$c^2 = 100 - 81$

Equations and Graphs of Ellipses

Using the distance formula, we can see that parabolas have the following equations: ***for a > b > 0,***

Horizontal	**Vertical**
$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$	$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$
Vertices: $(\pm a, 0)$ / Co-Vertices: $(0, \pm b)$	$(0, \pm a)$ / Co vert: $(\pm b, 0)$
Major Axis: Horizontal length 2a	Vertical length 2a
Minor Axis: Vertical length $2b$	Horizontal length $2b$
Foci: $(\pm c, 0)$, $c^2 = a^2 - b^2$	$(0, \pm c)$, $c^2 = a^2 - b^2$





Example

horiz.

An ellipse has the equation $\frac{x^2}{9} + \frac{y^2}{4} = 1$

6 4

Find the foci, vertices, and the lengths of the major and minor axes, and sketch the graph.

Vert: $(\pm a, 0) = (\pm 3, 0)$

Co Vert: $(0, \pm b) = (0, \pm 2)$

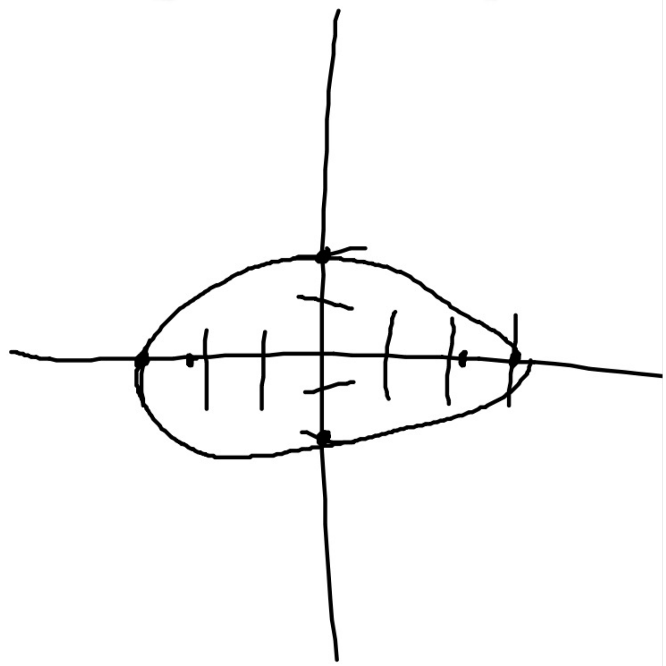
Foci: $(\pm c, 0) = (\pm\sqrt{5}, 0)$

$c^2 = a^2 - b^2$

$c^2 = 9 - 4$

$c^2 = 5$

$c = \pm\sqrt{5}$



Example

An ellipse has the equation $\frac{x^2}{36}+\frac{y^2}{25}=1$

12 10

Find the foci, vertices, and the lengths of the major and minor axes, and sketch the graph.

Vert.: $(\pm 6, 0)$

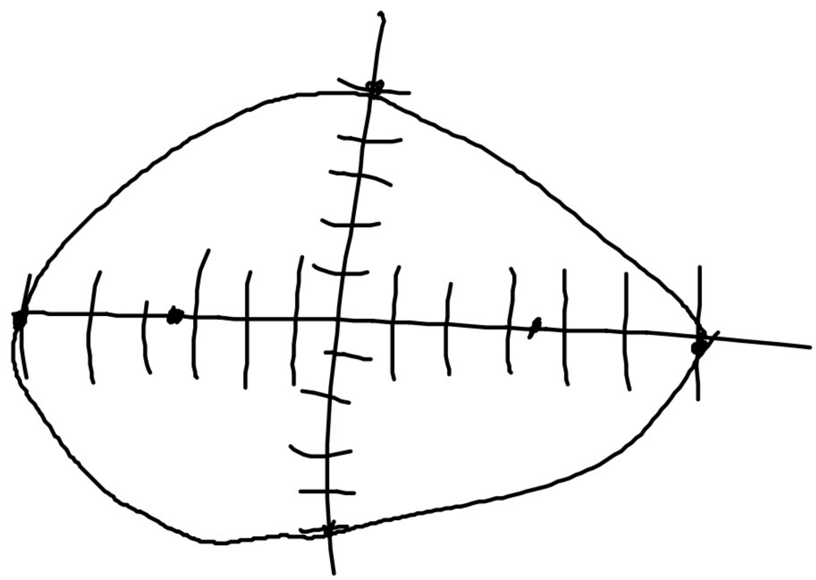
Covert: $(0, \pm 5)$

Foci: $(\pm\sqrt{11}, 0)$

$c^2=a^2-b^2$

$c^2=36-25$

$c=\pm\sqrt{11}$



$16x^2+9y^2=1$

$\frac{x^2}{1/16}+\frac{y^2}{1/9}=1$

Example

Find the foci of the ellipse $\frac{16x^2}{144}+\frac{9y^2}{144}=\frac{144}{144}$, and sketch its graph.

Vert: $(0, \pm 4)$

Covert: $(\pm 3, 0)$

$\Rightarrow \frac{x^2}{9}+\frac{y^2}{16}=1$

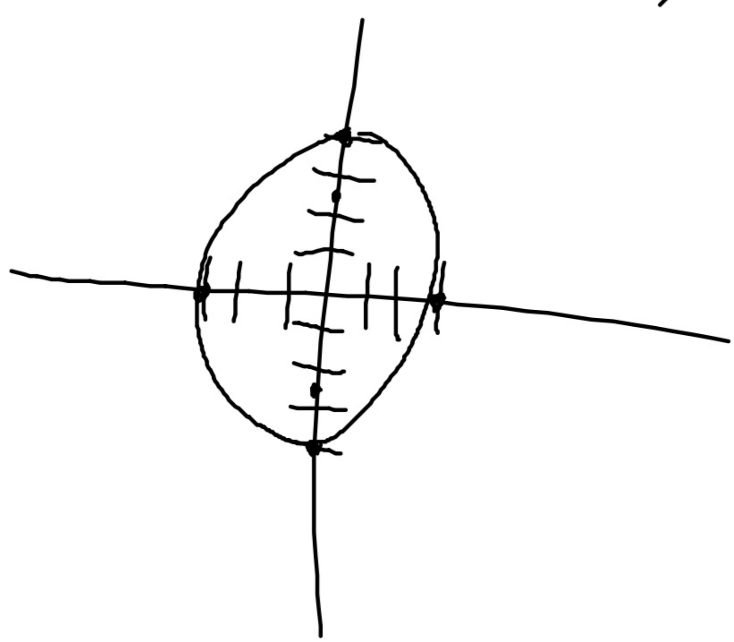
$c^2=a^2-b^2$

$c^2=16-9$

$c^2=7$

$c=\pm\sqrt{7}$

$F(0, \pm\sqrt{7})$



Example

horiz.

The vertices of an ellipse are $(\pm 4, 0)$ and foci are $(\pm 2, 0)$. Find its equation and sketch the graph.

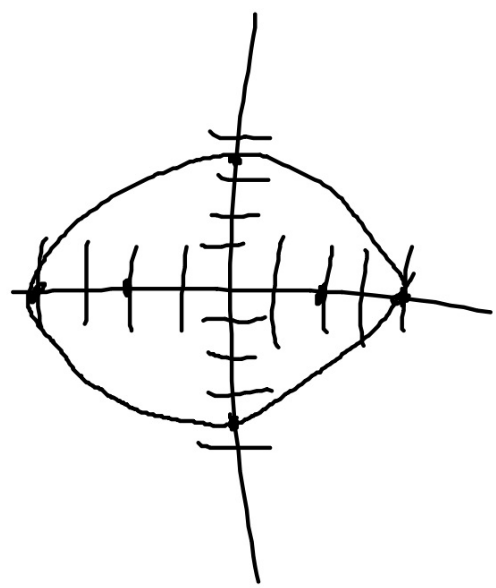
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \Rightarrow \frac{x^2}{16} + \frac{y^2}{12} = 1$$

$$c^2 = a^2 - b^2$$

$$4 = 16 - b^2$$

$$b^2 = 12$$

$$b = \pm\sqrt{12}$$



Homework 5/19

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