

Short Answer

$x$	0	1	2	3	4	5	6
$f(x)$	9.3	9.0	8.3	6.5	2.3	-7.6	-10.5

calculator permitted

6. The table above gives the values of a function obtained from an experiment. Use them to estimate

$$\int_0^6 f(x) dx \text{ using three equal subintervals using } \Delta x = \frac{6-0}{3} = 2$$

(a) right endpoints (REP)

$$\int_0^6 f(x) dx \approx 2 [8.3 + 2.3 - 10.5] = 0.2 = R_3$$

(b) left endpoints (LEP)

$$\int_0^6 f(x) dx \approx 2 [9.3 + 8.3 + 2.3] = 39.8 = L_3$$

(c) midpoints (MDPT)

$$\int_0^6 f(x) dx \approx 2 [9.0 + 6.5 - 7.6] = 15.8 = M_3$$

(d) the trapezoidal rule (TRAP)

$$\int_0^6 f(x) dx \approx \left(\frac{1}{2}\right)(2) [9.3 + (2)(8.3) + 2(2.3) - 10.5] = \frac{R_3 + L_3}{2} = 20 = T_3$$

(e) If the function is said to be a decreasing function, can you say whether your estimates are less than or greater than the exact value of the integral? Could any of these estimates approximate the area of the enclosed region with the  $x$ -axis? Why or why not?



$$L_3 > \int_0^6 f(x) dx$$

$$R_3 < \int_0^6 f(x) dx$$

$$T_3 > \int_0^6 f(x) dx$$



$$L_3 > \int_0^6 f(x) dx$$

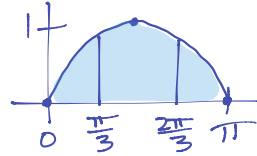
$$R_3 < \int_0^6 f(x) dx$$

$$T_3 < \int_0^6 f(x) dx$$

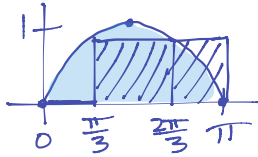
\*None can represent area approximations because  $f(x) < 0$  for some  $x$ -values  $\in [0, 6]$  (Area is POSITIVE!!)

7. Approximate the area of the region bounded by the graph of  $y = \sin x$  and the  $x$ -axis from  $x = 0$  to  $x = \pi$  using 3 equal subintervals using

$$\Delta x = \frac{\pi - 0}{3} = \frac{\pi}{3}$$

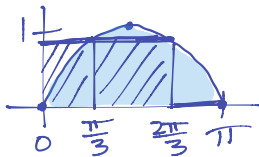


- (a) left endpoints



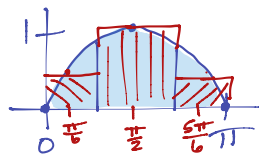
$$\begin{aligned} \text{Area} &\approx \frac{\pi}{3} (\sin 0 + \sin \frac{\pi}{3} + \sin \frac{2\pi}{3}) \\ &= \frac{\pi}{3} (0 + \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2}) \\ &= \frac{\pi}{3} \cdot \sqrt{3} = \frac{\sqrt{3}\pi}{3} = L_3 \end{aligned}$$

- (b) right endpoints



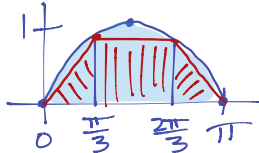
$$\begin{aligned} \text{Area} &\approx \frac{\pi}{3} [\sin \frac{\pi}{3} + \sin \frac{2\pi}{3} + \sin \pi] \\ &= \frac{\pi}{3} [\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} + 0] \\ &= \frac{\sqrt{3}\pi}{3} = R_3 \end{aligned}$$

- (c) midpoints



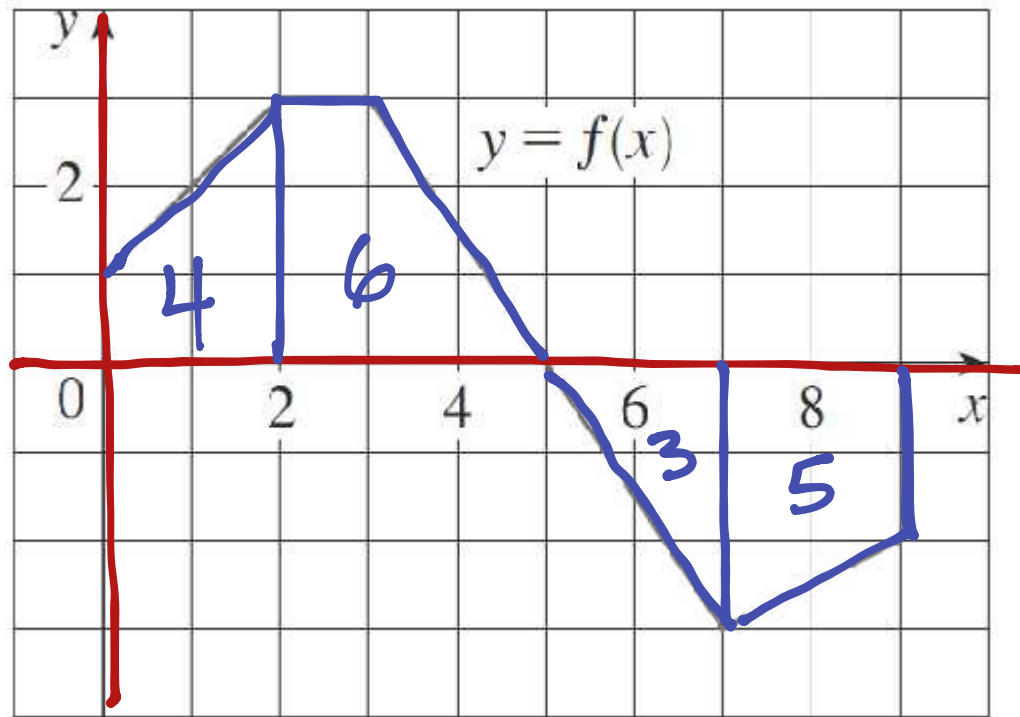
$$\begin{aligned} \text{Area} &\approx \frac{\pi}{3} [\sin \frac{\pi}{6} + \sin \frac{\pi}{2} + \sin \frac{5\pi}{6}] \\ &= \frac{\pi}{3} [\frac{1}{2} + 1 + \frac{1}{2}] \\ &= \frac{2\pi}{3} = M_3 \end{aligned}$$

- (d) trapezoidal rule



$$\begin{aligned} \text{Area} &\approx \left(\frac{1}{2}\right)\left(\frac{\pi}{3}\right) [\sin 0 + 2\sin \frac{\pi}{3} + 2\sin \frac{2\pi}{3} + \sin \pi] \\ &= \frac{\pi}{6} [0 + \sqrt{3} + \sqrt{3} + 0] \\ &= \frac{\sqrt{3}\pi}{3} = J_3 = \frac{R_3 + L_3}{2} \end{aligned}$$

8. The graph of  $f$  is shown below. Evaluate each integral by interpreting it in terms of areas.



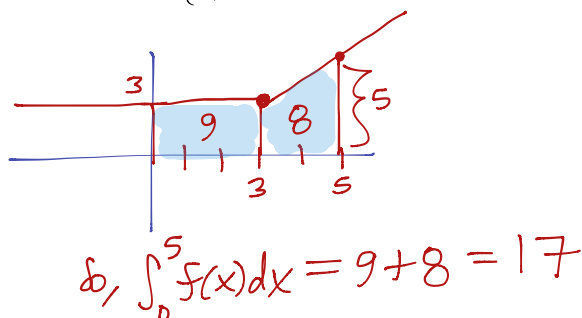
(a)  $\int_0^2 f(x) dx$   
4

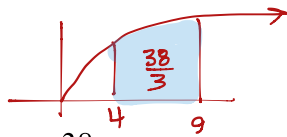
(b)  $\int_0^5 f(x) dx$   
4 + 6  
10

(c)  $\int_5^7 f(x) dx$   
-3

(d)  $\int_0^9 f(x) dx$   
4 + 6 - 3 - 5  
10 - 8  
2

9. Find  $\int_0^5 f(x) dx$  if  $f(x) = \begin{cases} 3, & x < 3 \\ x, & x \geq 3 \end{cases}$ . (Hint: Sketch the graph and interpret the areas)

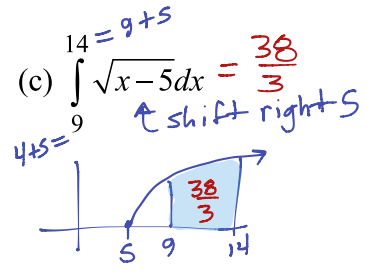




10. Given that  $\int_4^9 \sqrt{x} dx = \frac{38}{3}$ , using your knowledge of transformations, what is

(a)  $\int_9^4 \sqrt{t} dt$   
 $= -\int_4^9 \sqrt{t} dt$   
 $= -\frac{38}{3}$

(b)  $\int_4^9 (\sqrt{x} + 3) dx$   
 $= \int_4^9 \sqrt{x} dx + \int_4^9 3 dx$   
 $= \frac{38}{3} + 3(9-4)$   
 $= \frac{38}{3} + 15$   
 $= \frac{38 + 45}{3}$   
 $= \frac{83}{3}$



(d)  $\int_4^4 \sqrt{x} dx$   
 $= 0$   
 (haven't gone anywhere)  
 $* \int_a^a f(x) dx = 0$

calculator permitted

11. If  $f(x)$  is represented by the table below, approximate  $\int_1^{9.6} f(x) dx$  using left-endpoint, right-endpoint, midpoint, and trapezoidal approximations. Label each one. Use as many subintervals as the data allows.

		1.5		1.5		2		2		0.8		0.8		
$x$	1		2.5		4		6		8		8.8		9.6	<del>10.4</del>
$f(x)$	4		3		1		3		5		6		4	<del>7</del>

$\int_1^{9.6} f(x) dx \approx L_6 = (1.5)(4) + (1.5)(3) + (2)(1) + (2)(3) + (0.8)(5) + (0.8)(6)$   
 $= 27.3$

$\int_1^{9.6} f(x) dx \approx R_6 = (1.5)(3) + (1.5)(1) + 2(3) + 2(5) + (0.8)(6) + (0.8)(4)$   
 $= 30$

$\int_1^{9.6} f(x) dx \approx M_3 = (3)(3) + (4)(3) + (1.6)(6)$   
 $= 30.6$

$\int_1^{9.6} f(x) dx \approx T_6 = \frac{1}{2} [(1.5)(4+3) + (1.5)(3+1) + 2(1+3) + 2(3+5) + 0.8(5+6) + 0.8(6+4)]$   
 $= 28.65$