$\left(\frac{b}{2}\right)^2 = \left(\frac{-1}{2}\right)^2$

$= 1/4$

Warm Up 9/19

$\frac{-12}{4} - \frac{1}{4}$

1. Complete the square: $\frac{f(x)}{-1} = \frac{-x^2}{-1} \frac{+x}{-1} + \frac{3}{-1}$

$-f(x) = x^2 - x - 3$

$\Rightarrow -f(x) + \frac{1}{4} = (x^2 - x + 1/4) - 3 = \left(x - \frac{1}{2}\right)^2 - 3 - 1/4$

$-1/4$

$\Rightarrow \frac{-f(x)}{\cancel{-1}} = \left(x - \frac{1}{2}\right)^2 - 13/4$

$f(x) = -\left(x - \frac{1}{2}\right)^2 + 13/4$

Lesson 2-5 Part III

Quadratic Function: Maxima and Minima

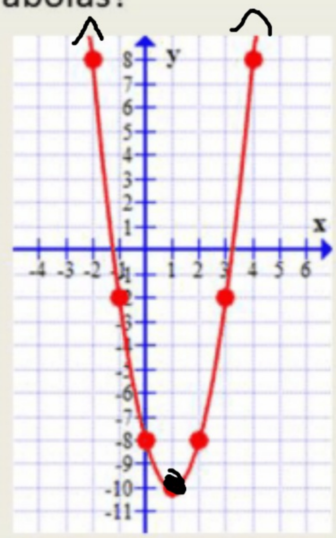
Objective

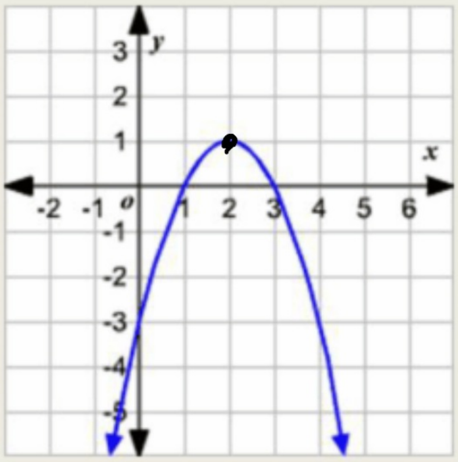
Students will…

- Be able to identify the vertex of a parabola as the maximum or the minimum value of the function.
- Be able to model and solve applicable word problems using quadratic functions.

Consider the following…

Do you see a maximum or minimum value in the following parabolas?





Maxima and Minima

Visually it's clear that the vertex of a parabola is naturally the maximum or a minimum, depending upon the orientation of the parabola. If the parabola opens down, the vertex is the maximum (at the highest point), while if graph opens up its vertex is the minimum (at the lowest point).

Ex.

Identify whether the function will have a minimum or a maximum.

Ex. $f(x) = x^2 - 6x + 8$ min.

Ex. $f(x) = -2x^2 - 3x + 9$ max

Examples

Find the maximum or minimum value of each quadratic function.

1. $f(x) = x^2 + 4x$ min.

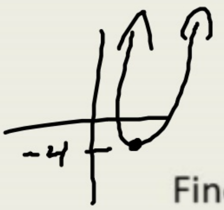
$$x = \frac{-b}{2a} = \frac{-4}{2} = -2$$

$$f(-2) = (-2)^2 + 4(-2) = 4 - 8 = \boxed{-4}$$

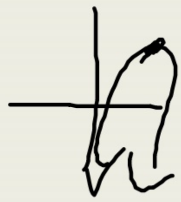
2. $g(x) = -2x^2 + 4x - 5$ max.

$$x = \frac{-b}{2a} = \frac{-4}{2(-2)} = 1$$

$$g(1) = -2(1)^2 + 4(1) - 5 = -2 + 4 - 5 = \boxed{-3}$$



Examples



Find the domain and the range of the function.

1. $f(x) = x^2 - 2x - 3$

D: $(-\infty, \infty)$

R: $[-4, \infty)$

$x = \frac{-b}{2a} = \frac{2}{2} = 1$

$f(1) = 1 - 2 - 3 = -4$

2. $f(x) = -x^2 + 4x - 3$

D: $(-\infty, \infty)$.

R: $(-\infty, 1]$

$x = \frac{-4}{2(-1)} = 2$

$f(2) = -4 + 8 - 3 = 1$

Most cars get their best gas mileage when traveling at a relatively modest speed. The gas mileage M for a certain new car is modeled by the function

$$M(s) \cancel{f(x)} = -\frac{1}{28}s^2 + 3s - 31,\ 15 \le s \le 70$$

Where s is the speed in mi/h and M is measured in mi/gal. What is the car's best gas mileage, and at what speed is it attained?

$$s = \frac{-b}{2a} = \frac{-3}{2\left(-\frac{1}{28}\right)} = \frac{3}{1/14} = 3 \cdot 14 = \boxed{42 \text{ mi/h}}$$

$$M(42) = -\frac{1}{28}(42)^2 + 3(42) - 31 = 32 \text{ mpg}$$

If a ball is thrown directly upward with a velocity of 50 ft/s, its height (in feet) after t seconds is given by

$$y = 50t - 12t^2 = -12t^2 + 50t$$

What is the maximum height attained by the ball?

$$t = \frac{-50}{2(-12)} = \frac{25}{12} \approx 2.08$$

$$50(2.08) - 12(2.08)^2 = 52.1 \text{ ft.}$$

After how many seconds did the ball reach its maximum height?

A soft-drink vendor at a popular beach analyzes his sales records, and finds that if he sells *x* cans of soda pop in one day, his profit (in dollars) is given by

$$P(x) = -0.001x^2 + 3x - 1800$$

What is his maximum profit per day, and how many cans must he sell for maximum profit?

$$x = \frac{-b}{2a} = \frac{-3}{(-0.001)(2)} = 1500$$

$$P(1500) = -0.001(1500)^2 + 3(1500) - 1800$$

$$= -2250 + 4500 - 1800$$

$$= \boxed{\$450}$$

A ball is thrown across a playing field. Its path is given by the equation,

$$y = -0.005x^2 + x + 5$$

where x is the distance the ball has traveled horizontally, and y is the height above ground level, both measured in feet.

a. What is the maximum height attained by the ball?

b. How far has it traveled horizontally when it hits the ground?

$$y = -0.005x^2 + x + 5$$

b. How far has it traveled horizontally when it hits the ground?

Homework 9/19

TB pg. 201-202 #29, 43, 49, 59, 61, 65, 71