Warm Up 9/11

Describe the shift of the function: $g(x) = (x + 11)^2 - 2$ from its "parent" function, $f(x) = x^2$

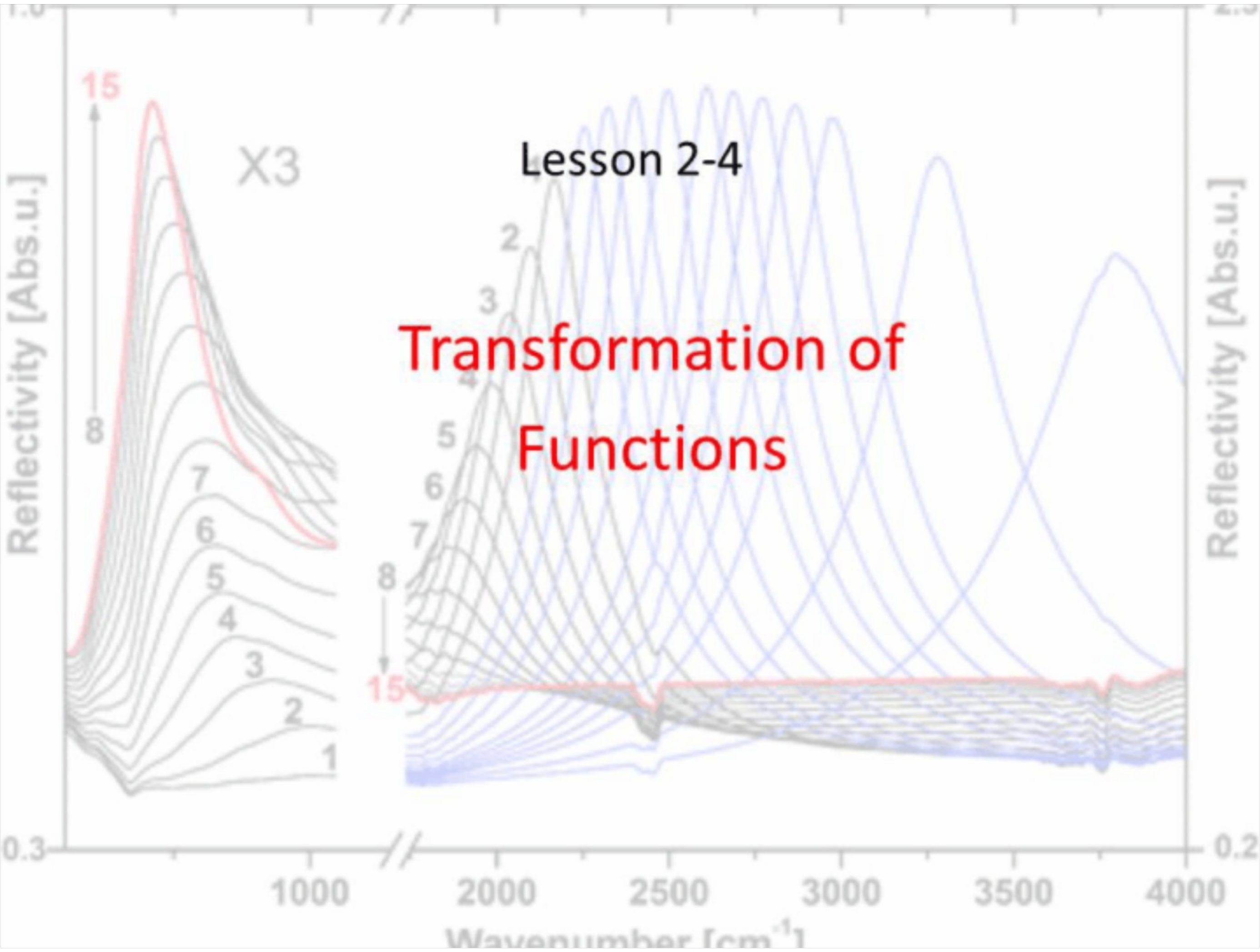
left 11, down 2

Describe the shift of the function $h(x) = (x - 6)^5 + 1$ from its "parent" function, $f(x) = x^5$

Right 6, up 1.

Lesson 2-4

Transformation of Functions



Objective

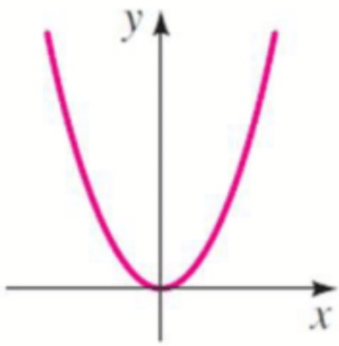
Students will…

- Be able to apply the properties of reflections in graphing various functions.
- Be able to determine whether a function is even or odd.

“Parent” Functions

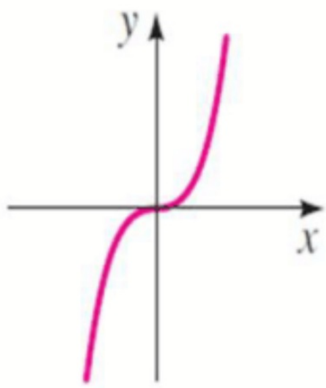
We have seen and studied some of the standard functions and their graphs. For example.

$f(x) = x^2$



$f(x) = x^2$

$f(x) = x^3$



$f(x) = x^3$

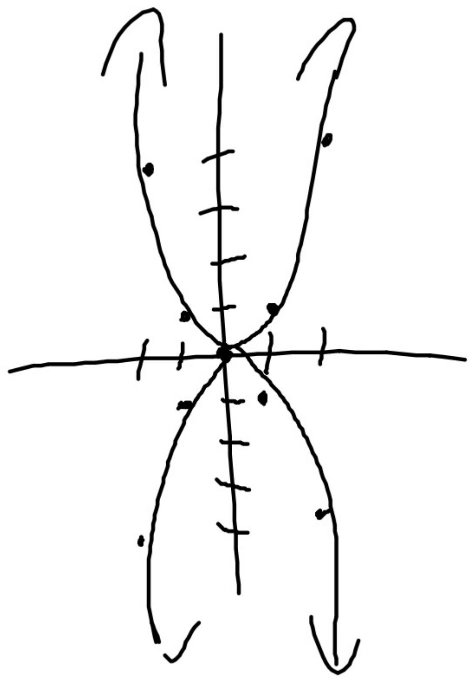
$-x^2 \neq (-x)^2$

Transformation of Functions

Let's go ahead and compare the two functions: $f(x) = x^2$ and $g(x) = -x^2$

x	x^2
-2	4
-1	1
0	0
1	1
2	4

x	$-x^2$
-2	-4
-1	-1
0	0
1	-1
2	-4

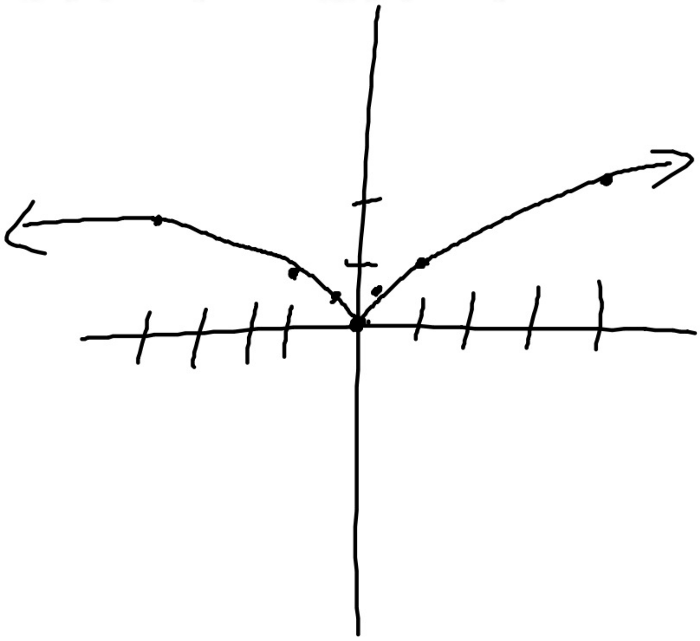


Transformation of Functions

Now let's compare the functions: $f(x) = \sqrt{x}$ and $g(x) = \sqrt{-x}$

x	$\sqrt{x}$
0	0
1/4	1/2
1	1
4	2

x	$\sqrt{-x}$
0	0
-1/4	1/2
-1	1
-4	2



Transformation: Reflection

As observed, the differences between the two functions were either **horizontal or vertical** reflection. This can be generalized by the following:

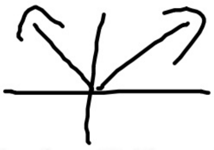
Along the y-axis (horizontal)

$y = f(-x)$ reflects the graph of $y = f(x)$ along the y-axis (horizontal reflection).

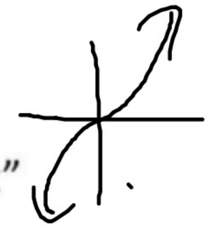
Along the x-axis (vertical)

$y = -f(x)$ reflects the graph of $y = f(x)$ along the x-axis (vertical reflection).

$|x|$

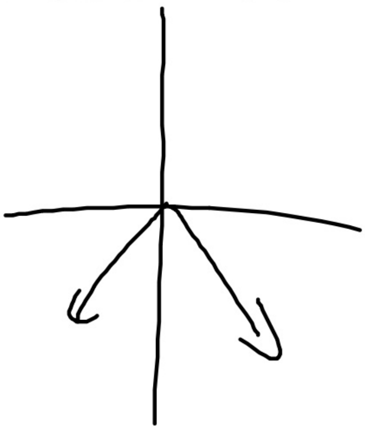


Examples

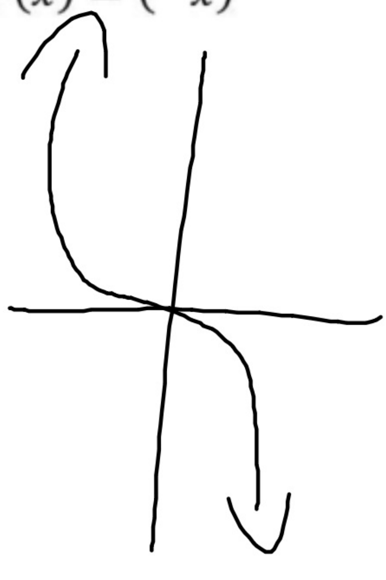


Sketch the following functions by transforming its “parent” function.

a. $f(x) = -|x|$



b. $f(x) = (-x)^3$



Even Functions

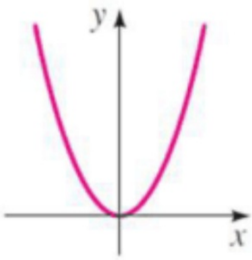
Consider the function $f(x) = x^2$. We observed that it can be reflected vertically, i.e. along the x-axis. What happens when we try to reflect this function horizontally, i.e. along the y-axis?

This would mean that the equation would be written in the form of

$$f(x) = (-x)^2 = x^2$$

So, as you can see horizontal reflection really did not change anything. This can easily be seen on its graph.

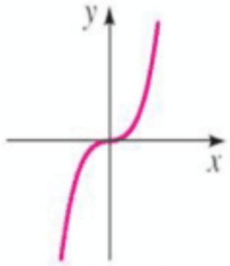
Any function that has this characteristic is called an **<u>even</u>** function.



$f(x) = x^2$

Odd Functions

Now consider the function $f(x) = x^3$. We have already seen it reflected horizontally, i.e. along the y-axis. What happens when we reflect this graph vertically, i.e. along the x-axis? Look at the graph!



$f(x) = x^3$

Here the graph looks the same whether it is reflected vertically or horizontally. This can easily be seen algebraically: $(-x)^3 = -x^3$

$(-x)^3 = -(x^3)$.

Any function that has this characteristic is called an **odd** function.

Even and Odd Functions

So now we give a formal, generalized definition of even and odd functions:

Let f be a function,

f is even if $f(-x) = f(x)$, for all x in the domain of f

f is odd if $f(-x) = -(f(x))$ for all x in the domain of f

Ex. Determine whether the following functions are even or odd.

a. $f(x) = x^5 + x$

$f(-x) = (-x)^5 + (-x)$

$= -x^5 - x$

$= -(x^5 + x)$

$= -f(x)$

Odd

b. $g(x) = 1 - x^4$

$g(-x) = 1 - (-x)^4$

$= 1 - x^4$

$= g(x)$

even

c. $h(x) = 2x - x^2$

$h(-x) = 2(-x) - (-x)^2$

$= -2x - x^2$

$= -(2x + x^2)$

Neither

Homework 9/11

TB pg. 190 #16, 35, 36, 40, 61-68