Warm Up 9/4

Let $f(x) = 2x^2 + 3x - 1$. Evaluate $f(a)$, $f(a+h)$, $f = \frac{f(a+h)-f(a)}{h}$

$f(a) = 2a^2 + 3a - 1$

$f(a+h) = 2(a+h)^2 + 3(a+h) - 1 \Rightarrow$

$= 2(a^2 + 2ah + h^2) + 3a + 3h - 1$

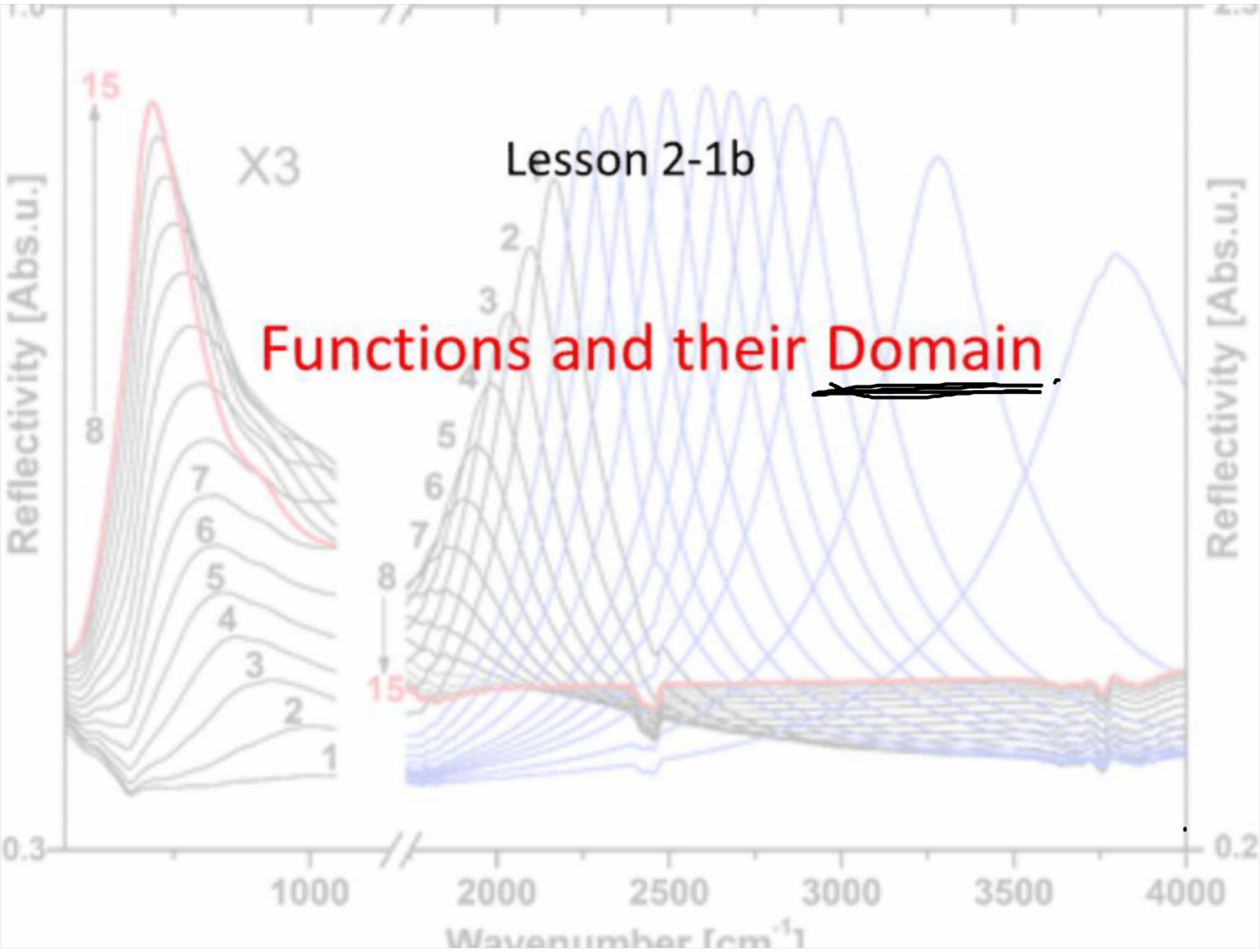
$= 2a^2 + 4ah + 2h^2 + 3a + 3h - 1$

$\frac{2a^2 + 4ah + 2h^2 + 3a + 3h - 1 - (2a^2 + 3a - 1)}{h} = \frac{4ah + 2h^2 + 3h}{h}$

$= \frac{h(4a + 2h + 3)}{h} = \boxed{4a + 2h + 3}$

Lesson 2-1b

Functions and their Domain



Objective

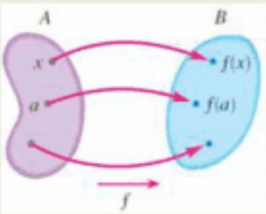
Students will…

- Be able to solve word problems using functional relationship.
- Be able to find the domains of functions.
- Be able to represent functions in multiple ways.

Definition of a Function

So now we are ready to define what a function is.

A **function**, say f, is a rule that assigns to each element (item) x in a certain set A **exactly one** element, called $f(x)$, in a set B.

Ex. 

Another way to define function is for every **input**, there is exactly **one** **output**.

The set A is also known as the **domain**, and set B is known as the **range**.

Word Problems Using Functions

If an astronaut weighs 130 pounds on the surface of the earth, then her weight when she is h miles above the earth is given by the function: $w(h) = 130\left(\frac{3960}{3960+h}\right)^2$

a. What is her weight when she is 100 mi above the earth?

$$w(100) = 130\left(\frac{3960}{3960+100}\right)^2 = 130\left(\frac{3960}{4060}\right)^2 \approx 123.66$$

b. Construct a table of values of the function w that gives her weight at heights from 0 to 500 mi. What do you conclude from the table?

h	$w(h)$
0	130
100	123.6
200	117.8
⋮	
500	102.5

Word Problems Using Functions

If the speed limit on a 100-mile stretch of road is 75 miles per hour, then the amount of time it takes a car going x miles per hour over the limit to travel the stretch is given by $f(x) = \frac{100}{75+x}$

a. How long does it take the car to travel the stretch if the car is going 10 miles per hour over the limit?

$f(10) = \frac{100}{75+10} \approx \frac{100}{85} \approx 1.18$ hrs

b. How long does it take the car to travel the stretch if the car is not speeding at all?

$f(0) = \frac{100}{75} \approx 1.33$ hrs.

Domain of a Function

Recall that the **domain** of a function is the set of all **inputs**. Domain may be written **explicitly**. For example, for the function $f(x) = x^2, \quad 0 \leq x \leq 5$, the domain is specifically set as all inputs between and including 0 and 5. Hence its domain is simply $[0, 5]$.

Whenever we have a function without the domain stated explicitly, we need to figure it out by algebraic reasoning.

Ex. $f(x) = x^2 + 1$

$f(0) = 1$

$(-\infty, \infty)$

$\{x : \mathbb{R}\}$

$g(x) = \frac{1}{x-4}$

$\{x : x \neq 4\}$

$x - 4 = 0$

$x = 4$

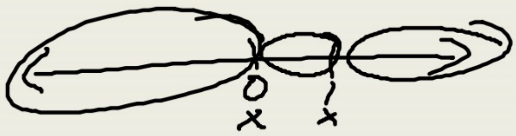
$(-\infty, 4) \cup (4, \infty)$

$h(x) = \sqrt{x}$

$\{x : x \geq 0\}$

$x \geq 0$

$[0, \infty)$



Examples

$\{x : -3 \le x \le 3\}$.

Find the domain of each function.

a. $f(x) = \frac{1}{x^2 - x}$

$x^2 - x = 0$ $(-\infty, 0) \cup (0,1) \cup (1, \infty)$.

$x(x-1) = 0$

$x = 0$

$x = 1$

b. $g(x) = \sqrt{9 - x^2}$ $[-3, 3]$.

$9 - x^2 \ge 0$
$+x^2 \quad +x^2$

$\sqrt{x^2} \le \sqrt{9} \Rightarrow x \le \pm 3$.

$x \le 3$, $x \ge -3$.

c. $h(t) = \frac{t}{\sqrt{t+1}}$

$t + 1 > 0$ $(-1, \infty)$
$-1 \quad -1$

$t > -1$ $\{t : t > -1\}$

Four Ways of Representing a Function

To help us understand what a function is, we have used machine and arrow diagrams. We can represent a functional relationship in following ways:

1. Verbally (by a description in words)

2. Algebraically (by an explicit formula)

3. Visually (by a graph)

4. Numerically (by a table of values)

Four Ways to Represent a Function

Four Ways to Represent a Function

Verbal

Using words:

$P(t)$ is "the population of the world at time t"

Relation of population P and time t

Algebraic

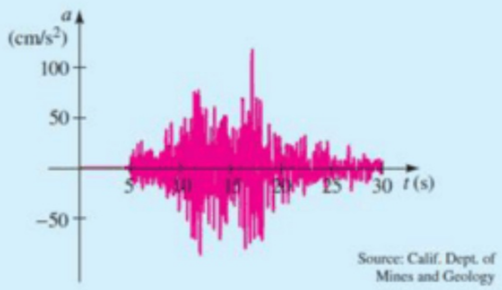
Using a formula:

$$A(r) = \pi r^2$$

Area of a circle

Visual

Using a graph:



Vertical acceleration during an earthquake

Numerical

Using a table of values:

w (ounces)	$C(w)$ (dollars)
$0 < w \leq 1$	0.37
$1 < w \leq 2$	0.60
$2 < w \leq 3$	0.83
$3 < w \leq 4$	1.06
$4 < w \leq 5$	1.29
⋮	⋮

Cost of mailing a first-class letter

Homework 9/4

TB pg. 156 #34, 38, 42, 44, 50, 58, 59, 66