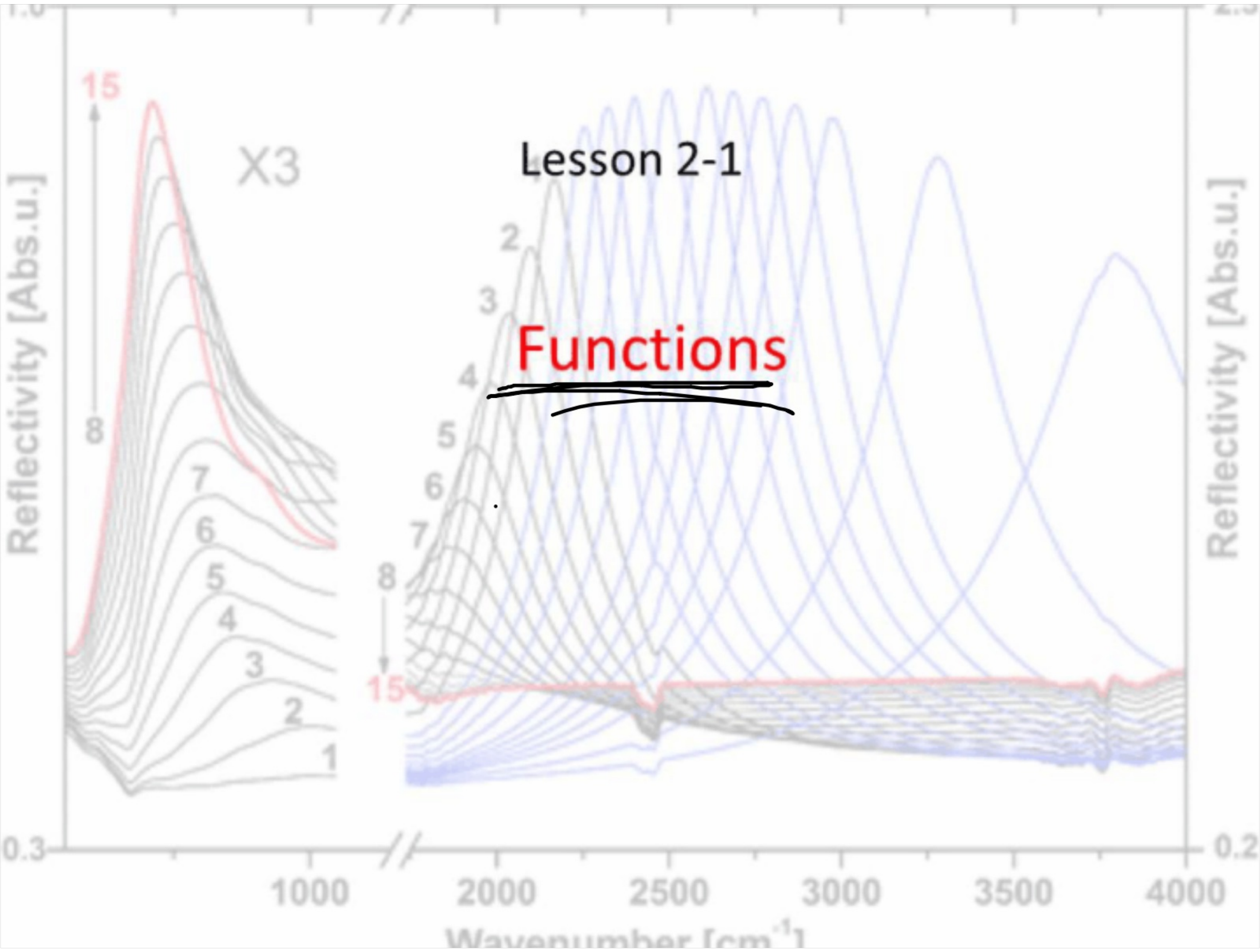
Lesson 2-1

Functions



Objective

Students will…

- Be able to define what an input and an output is.
- Be able to define what a function is.

Functional Relationship

A **functional relationship** is a relationship in which one quantity **depends** on another. In other words, given two variables, one is always **dependent** on the other.

ex. Height is a function of age

Temperature is a function of date

Cost of mail is a function of weight.

Independent vs Dependent Variables

That being said, we must always be able to define both the independent and dependent variables.

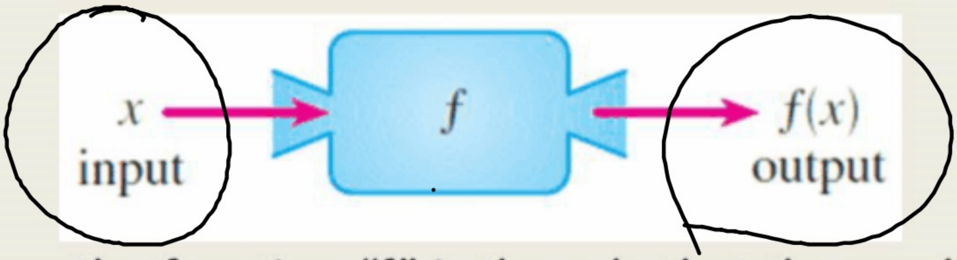
ex. Height is a function of age.

Temperature is a function of date.

Cost of mail is a function of weight.

Input vs Output

Mathematically speaking, we can also differentiate the independent and the dependent variables as inputs and outputs. Consider the following picture:



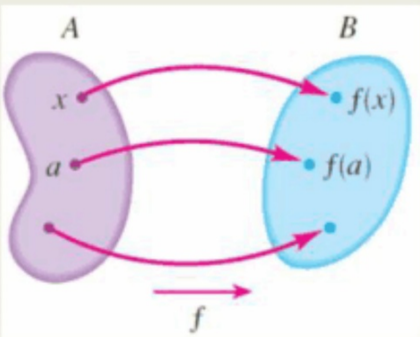
Here the function "f" is the rule that the machine operates in, and what comes out depends on what goes in.

Definition of a Function

So now we are ready to define what a function is.

A **function**, say f, is a rule that assigns to each element (item) x in a certain set A **exactly one** element, called $f(x)$, in a set B.

Ex.



$f : A \longrightarrow B$

The set A is also known as the **domain**, and set B is known as the **range**.

Examples of Functions

Another way to define function is for every **input**, there is exactly **<u>one</u>** **output**.

Ex.

$f(x) = x - 3$

x	$f(x)$
-1	-4
0	-3
1	-2

$f(x) = x^2$

x	$f(x)$
-1	1
0	0
1	1

$f(x) = \sin x$

$\sqrt{x} = \sqrt{y^2}$

$y = \boxed{\pm}\sqrt{x}$

x	y
4	$2, -2$

Not a function

Evaluating Functions

Consider the function $f(x) = x - 3$

Here, x is the input, while $f(x)$ is the output. That being said, $f(x)$ would change as x changes. We can evaluate functions by placing different inputs. For the above function,

$$f(1) = (1) - 3 = -2$$
$$f(2) = (2) - 3 = -1$$
$$f(0) = (0) - 3 = -3$$
$$f(-3) = (-3) - 3 = -6$$

Examples

Let $f(x) = 3x^2 + x - 5$. Evaluate each function value.

1. $f(-2)$

$f(-2) = 3(-2)^2 + (-2) - 5$

$12 - 2 - 5 = 5$

2. $f(0)$

$f(0) = -5$

3. $f(4)$

$f(4) = 47$

4. $f(\frac{1}{2})$

$f(1/2) = -3.75$

$3(1/4) = 3/4 + 1/2 - \frac{5}{1}$

$3/4 + 2/4 - \frac{20}{4}$

$= \frac{-15}{4} = -3\ 3/4$

Piecewise Functions

Piecewise functions are combination of functions that are defined by the range of inputs.

Ex.
$$C(x) = \begin{cases} 39 & \text{if } 0 \leq x \leq 400 \\ 39 + 0.2(x - 400) & \text{if } x > 400 \end{cases}$$

$C(398) = 39$ $C(401) = 39.2$ $C(-2) =$ Und.

So whenever x is in between or equal to 0 and 400, then the output is always 39. Whenever x is strictly above 400, the bottom function applies.

Examples

Evaluate.

22. $f(x) = \begin{cases} 5 & \text{if } x \le 2 \\ 2x - 3 & \text{if } x > 2 \end{cases}$

$f(-3), f(0), f(2), f(3), f(5)$

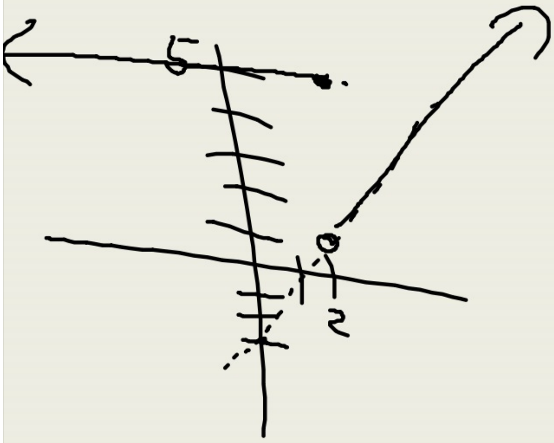
$f(-3) = 5$

$f(0) = 5$

$f(2) = 5$

$f(3) = 6 - 3 = 3$

$f(5) = 10 - 3 = 7$



Examples

Use the function to evaluate the indicated expression.

$f(x) = 3x - 1; f(2x), 2f(x)$

$f(2x) = 3(2x) - 1 = 6x - 1$

$2f(x) = 2(3x-1) = 6x - 2$

Examples

Find $f(a)$, $f(a+h)$, and the difference quotient $\frac{f(a+h)-f(a)}{h}$

$(a+h)(a+h)$ FOIL

$f(x) = x^2 + 1$

$f(a) = a^2+1$

$f(a+h) = (a+h)^2+1 = a^2+2ah+h^2+1$

$$\frac{\cancel{a^2}+2ah+h^2\cancel{+1}-(\cancel{a^2}\cancel{+1})}{h} = \frac{2ah+h^2}{h} = \frac{\cancel{h}(2a+h)}{\cancel{h}} = \boxed{2a+h}$$

Homework 9/3

TB pg. 155 #1-4, 12, 16, 24, 28, 34