1.

Row Operation 1:	$\begin{bmatrix} -1 & 1 \\ -1 & 0 \\ 0 & -1 \\ -1 & -2 \end{bmatrix}$	multiply the 1st row by -1	$\begin{bmatrix} 1 & -1 \\ -1 & 0 \\ 0 & -1 \\ -1 & -2 \end{bmatrix}$
Row Operation 2:	$\begin{bmatrix} 1 & -1 \\ -1 & 0 \\ 0 & -1 \\ -1 & -2 \end{bmatrix}$	add 1 times the 1st row to the 2nd row	$\begin{bmatrix} 1 & -1 \\ 0 & -1 \\ 0 & -1 \\ -1 & -2 \end{bmatrix}$
Row Operation 3:	$\begin{bmatrix} 1 & -1 \\ 0 & -1 \\ 0 & -1 \\ -1 & -2 \end{bmatrix}$	add 1 times the 1st row to the 4th row	$\begin{bmatrix} 1 & -1 \\ 0 & -1 \\ 0 & -1 \\ 0 & -3 \end{bmatrix}$
Row Operation 4:	$\begin{bmatrix} 1 & -1 \\ 0 & -1 \\ 0 & -1 \\ 0 & -3 \end{bmatrix}$	multiply the 2nd row by -1	$\begin{bmatrix} 1 & -1 \\ 0 & 1 \\ 0 & -1 \\ 0 & -3 \end{bmatrix}$
Row Operation 5:	$\begin{bmatrix} 1 & -1 \\ 0 & 1 \\ 0 & -1 \\ 0 & -3 \end{bmatrix}$	add 1 times the 2nd row to the 3rd row	$\begin{bmatrix} 1 & -1 \\ 0 & 1 \\ 0 & 0 \\ 0 & -3 \end{bmatrix}$
Row Operation 6:	$\begin{bmatrix} 1 & -1 \\ 0 & 1 \\ 0 & 0 \\ 0 & -3 \end{bmatrix}$	add 3 times the 2nd row to the 4th row	$\begin{bmatrix} 1 & -1 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$

2.

Row Operation 1:	$\begin{bmatrix} -1 & 1 \\ -1 & 2 \\ -3 & 2 \end{bmatrix}$	multiply the 1st row by -1	$\begin{bmatrix} 1 & -1 \\ -1 & 2 \\ -3 & 2 \end{bmatrix}$
Row Operation 2:	$\begin{bmatrix} 1 & -1 \\ -1 & 2 \\ -3 & 2 \end{bmatrix}$	add 1 times the 1st row to the 2nd row	$\begin{bmatrix} 1 & -1 \\ 0 & 1 \\ -3 & 2 \end{bmatrix}$
Row Operation 3:	$\begin{bmatrix} 1 & -1 \\ 0 & 1 \\ -3 & 2 \end{bmatrix}$	add 3 times the 1st row to the 3rd row	$\begin{bmatrix} 1 & -1 \\ 0 & 1 \\ 0 & -1 \end{bmatrix}$
Row Operation 4:	$\begin{bmatrix} 1 & -1 \\ 0 & 1 \\ 0 & -1 \end{bmatrix}$	add 1 times the 2nd row to the 3rd row	$\begin{bmatrix} 1 & -1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$

3.

Row Operation 1:	$\begin{bmatrix} 1 & 3 & 2 & 1 \\ 2 & -3 & 0 & -2 \end{bmatrix}$	add -2 times the 1st row to the 2nd row	$\begin{bmatrix} 1 & 3 & 2 & 1 \\ 0 & -9 & -4 & -4 \end{bmatrix}$
Row Operation 2:	$\begin{bmatrix} 1 & 3 & 2 & 1 \\ 0 & -9 & -4 & -4 \end{bmatrix}$	multiply the 2nd row by -1/9	$\begin{bmatrix} 1 & 3 & 2 & 1 \\ 0 & 1 & \frac{4}{9} & \frac{4}{9} \end{bmatrix}$

4.

Row Operation 1:	$\begin{bmatrix} -1 & -3 \\ 3 & -3 \\ -3 & -3 \\ 2 & 0 \end{bmatrix}$	multiply the 1st row by -1	$\begin{bmatrix} 1 & 3 \\ 3 & -3 \\ -3 & -3 \\ 2 & 0 \end{bmatrix}$
Row Operation 2:	$\begin{bmatrix} 1 & 3 \\ 3 & -3 \\ -3 & -3 \\ 2 & 0 \end{bmatrix}$	add -3 times the 1st row to the 2nd row	$\begin{bmatrix} 1 & 3 \\ 0 & -12 \\ -3 & -3 \\ 2 & 0 \end{bmatrix}$
Row Operation 3:	$\begin{bmatrix} 1 & 3 \\ 0 & -12 \\ -3 & -3 \\ 2 & 0 \end{bmatrix}$	add 3 times the 1st row to the 3rd row	$\begin{bmatrix} 1 & 3 \\ 0 & -12 \\ 0 & 6 \\ 2 & 0 \end{bmatrix}$
Row Operation 4:	$\begin{bmatrix} 1 & 3 \\ 0 & -12 \\ 0 & 6 \\ 2 & 0 \end{bmatrix}$	add -2 times the 1st row to the 4th row	$\begin{bmatrix} 1 & 3 \\ 0 & -12 \\ 0 & 6 \\ 0 & -6 \end{bmatrix}$
Row Operation 5:	$\begin{bmatrix} 1 & 3 \\ 0 & -12 \\ 0 & 6 \\ 0 & -6 \end{bmatrix}$	multiply the 2nd row by -1/12	$\begin{bmatrix} 1 & 3 \\ 0 & 1 \\ 0 & 6 \\ 0 & -6 \end{bmatrix}$
Row Operation 6:	$\begin{bmatrix} 1 & 3 \\ 0 & 1 \\ 0 & 6 \\ 0 & -6 \end{bmatrix}$	add -6 times the 2nd row to the 3rd row	$\begin{bmatrix} 1 & 3 \\ 0 & 1 \\ 0 & 0 \\ 0 & -6 \end{bmatrix}$
Row Operation 7:	$\begin{bmatrix} 1 & 3 \\ 0 & 1 \\ 0 & 0 \\ 0 & -6 \end{bmatrix}$	add 6 times the 2nd row to the 4th row	$\begin{bmatrix} 1 & 3 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$

5.

Row Operation 1:	$\begin{bmatrix} 1 & 3 \\ -1 & 3 \\ 3 & 0 \\ 1 & -3 \end{bmatrix}$	add 1 times the 1st row to the 2nd row	$\begin{bmatrix} 1 & 3 \\ 0 & 6 \\ 3 & 0 \\ 1 & -3 \end{bmatrix}$
Row Operation 2:	$\begin{bmatrix} 1 & 3 \\ 0 & 6 \\ 3 & 0 \\ 1 & -3 \end{bmatrix}$	add -3 times the 1st row to the 3rd row	$\begin{bmatrix} 1 & 3 \\ 0 & 6 \\ 0 & -9 \\ 1 & -3 \end{bmatrix}$
Row Operation 3:	$\begin{bmatrix} 1 & 3 \\ 0 & 6 \\ 0 & -9 \\ 1 & -3 \end{bmatrix}$	add -1 times the 1st row to the 4th row	$\begin{bmatrix} 1 & 3 \\ 0 & 6 \\ 0 & -9 \\ 0 & -6 \end{bmatrix}$
Row Operation 4:	$\begin{bmatrix} 1 & 3 \\ 0 & 6 \\ 0 & -9 \\ 0 & -6 \end{bmatrix}$	multiply the 2nd row by 1/6	$\begin{bmatrix} 1 & 3 \\ 0 & 1 \\ 0 & -9 \\ 0 & -6 \end{bmatrix}$
Row Operation 5:	$\begin{bmatrix} 1 & 3 \\ 0 & 1 \\ 0 & -9 \\ 0 & -6 \end{bmatrix}$	add 9 times the 2nd row to the 3rd row	$\begin{bmatrix} 1 & 3 \\ 0 & 1 \\ 0 & 0 \\ 0 & -6 \end{bmatrix}$
Row Operation 6:	$\begin{bmatrix} 1 & 3 \\ 0 & 1 \\ 0 & 0 \\ 0 & -6 \end{bmatrix}$	add 6 times the 2nd row to the 4th row	$\begin{bmatrix} 1 & 3 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$

6.

Row Operation 1:	$\begin{bmatrix} 1 & 2 & 0 \\ 1 & 3 & 3 \\ -1 & 0 & -1 \\ -3 & 0 & 0 \end{bmatrix}$	add -1 times the 1st row to the 2nd row	$\begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 3 \\ -1 & 0 & -1 \\ -3 & 0 & 0 \end{bmatrix}$
Row Operation 2:	$\begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 3 \\ -1 & 0 & -1 \\ -3 & 0 & 0 \end{bmatrix}$	add 1 times the 1st row to the 3rd row	$\begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 3 \\ 0 & 2 & -1 \\ -3 & 0 & 0 \end{bmatrix}$
Row Operation 3:	$\begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 3 \\ 0 & 2 & -1 \\ -3 & 0 & 0 \end{bmatrix}$	add 3 times the 1st row to the 4th row	$\begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 3 \\ 0 & 2 & -1 \\ 0 & 6 & 0 \end{bmatrix}$
Row Operation 4:	$\begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 3 \\ 0 & 2 & -1 \\ 0 & 6 & 0 \end{bmatrix}$	add -2 times the 2nd row to the 3rd row	$\begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 3 \\ 0 & 0 & -7 \\ 0 & 6 & 0 \end{bmatrix}$
Row Operation 5:	$\begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 3 \\ 0 & 0 & -7 \\ 0 & 6 & 0 \end{bmatrix}$	add -6 times the 2nd row to the 4th row	$\begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 3 \\ 0 & 0 & -7 \\ 0 & 0 & -18 \end{bmatrix}$
Row Operation 6:	$\begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 3 \\ 0 & 0 & -7 \\ 0 & 0 & -18 \end{bmatrix}$	multiply the 3rd row by -1/7	$\begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \\ 0 & 0 & -18 \end{bmatrix}$
Row Operation 7:	$\begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \\ 0 & 0 & -18 \end{bmatrix}$	add 18 times the 3rd row to the 4th row	$\begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

7.

Row Operation 1:	$\begin{bmatrix} 3 & -3 & 2 \\ -2 & 2 & -2 \\ 2 & 1 & 1 \\ 3 & 1 & 1 \end{bmatrix}$	multiply the 1st row by 1/3	$\begin{bmatrix} 1 & -1 & \frac{2}{3} \\ -2 & 2 & -2 \\ 2 & 1 & 1 \\ 3 & 1 & 1 \end{bmatrix}$
Row Operation 2:	$\begin{bmatrix} 1 & -1 & \frac{2}{3} \\ -2 & 2 & -2 \\ 2 & 1 & 1 \\ 3 & 1 & 1 \end{bmatrix}$	add 2 times the 1st row to the 2nd row	$\begin{bmatrix} 1 & -1 & \frac{2}{3} \\ 0 & 0 & \frac{-2}{3} \\ 2 & 1 & 1 \\ 3 & 1 & 1 \end{bmatrix}$
Row Operation 3:	$\begin{bmatrix} 1 & -1 & \frac{2}{3} \\ 0 & 0 & \frac{-2}{3} \\ 2 & 1 & 1 \\ 3 & 1 & 1 \end{bmatrix}$	add -2 times the 1st row to the 3rd row	$\begin{bmatrix} 1 & -1 & \frac{2}{3} \\ 0 & 0 & \frac{-2}{3} \\ 0 & 3 & \frac{-1}{3} \\ 3 & 1 & 1 \end{bmatrix}$
Row Operation 4:	$\begin{bmatrix} 1 & -1 & \frac{2}{3} \\ 0 & 0 & \frac{-2}{3} \\ 0 & 3 & \frac{-1}{3} \\ 3 & 1 & 1 \end{bmatrix}$	add -3 times the 1st row to the 4th row	$\begin{bmatrix} 1 & -1 & \frac{2}{3} \\ 0 & 0 & \frac{-2}{3} \\ 0 & 3 & \frac{-1}{3} \\ 0 & 4 & -1 \end{bmatrix}$
Row Operation 5:	$\begin{bmatrix} 1 & -1 & \frac{2}{3} \\ 0 & 0 & \frac{-2}{3} \\ 0 & 3 & \frac{-1}{3} \\ 0 & 4 & -1 \end{bmatrix}$	interchange the 2nd row and the 3rd row	$\begin{bmatrix} 1 & -1 & \frac{2}{3} \\ 0 & 3 & \frac{-1}{3} \\ 0 & 0 & \frac{-2}{3} \\ 0 & 4 & -1 \end{bmatrix}$
Row Operation 6:	$\begin{bmatrix} 1 & -1 & \frac{2}{3} \\ 0 & 3 & \frac{-1}{3} \\ 0 & 0 & \frac{-2}{3} \\ 0 & 4 & -1 \end{bmatrix}$	multiply the 2nd row by 1/3	$\begin{bmatrix} 1 & -1 & \frac{2}{3} \\ 0 & 1 & \frac{-1}{9} \\ 0 & 0 & \frac{-2}{3} \\ 0 & 4 & -1 \end{bmatrix}$
Row Operation 7:	$\begin{bmatrix} 1 & -1 & \frac{2}{3} \\ 0 & 1 & \frac{-1}{9} \\ 0 & 0 & \frac{-2}{3} \\ 0 & 4 & -1 \end{bmatrix}$	add -4 times the 2nd row to the 4th row	$\begin{bmatrix} 1 & -1 & \frac{2}{3} \\ 0 & 1 & \frac{-1}{9} \\ 0 & 0 & \frac{-2}{3} \\ 0 & 0 & \frac{-5}{9} \end{bmatrix}$
Row Operation 8:	$\begin{bmatrix} 1 & -1 & \frac{2}{3} \\ 0 & 1 & \frac{-1}{9} \\ 0 & 0 & \frac{-2}{3} \\ 0 & 0 & \frac{-5}{9} \end{bmatrix}$	multiply the 3rd row by -3/2	$\begin{bmatrix} 1 & -1 & \frac{2}{3} \\ 0 & 1 & \frac{-1}{9} \\ 0 & 0 & 1 \\ 0 & 0 & \frac{-5}{9} \end{bmatrix}$
Row Operation 9:	$\begin{bmatrix} 1 & -1 & \frac{2}{3} \\ 0 & 1 & \frac{-1}{9} \\ 0 & 0 & 1 \\ 0 & 0 & \frac{-5}{9} \end{bmatrix}$	add 5/9 times the 3rd row to the 4th row	$\begin{bmatrix} 1 & -1 & \frac{2}{3} \\ 0 & 1 & \frac{-1}{9} \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

8.

Row Operation 1: multiply the 1st row by **-1**

$$\begin{bmatrix} -1 & 1 & 0 & 1 \\ -2 & -3 & -1 & -2 \\ -3 & -1 & -2 & -1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -1 & 0 & -1 \\ -2 & -3 & -1 & -2 \\ -3 & -1 & -2 & -1 \end{bmatrix}$$

Row Operation 2: add **2** times the 1st row to the 2nd row

$$\begin{bmatrix} 1 & -1 & 0 & -1 \\ -2 & -3 & -1 & -2 \\ -3 & -1 & -2 & -1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -1 & 0 & -1 \\ 0 & -5 & -1 & -4 \\ -3 & -1 & -2 & -1 \end{bmatrix}$$

Row Operation 3: add **3** times the 1st row to the 3rd row

$$\begin{bmatrix} 1 & -1 & 0 & -1 \\ 0 & -5 & -1 & -4 \\ -3 & -1 & -2 & -1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -1 & 0 & -1 \\ 0 & -5 & -1 & -4 \\ 0 & -4 & -2 & -4 \end{bmatrix}$$

Row Operation 4: multiply the 2nd row by **-1/5**

$$\begin{bmatrix} 1 & -1 & 0 & -1 \\ 0 & -5 & -1 & -4 \\ 0 & -4 & -2 & -4 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -1 & 0 & -1 \\ 0 & 1 & \frac{1}{5} & \frac{4}{5} \\ 0 & -4 & -2 & -4 \end{bmatrix}$$

Row Operation 5: add **4** times the 2nd row to the 3rd row

$$\begin{bmatrix} 1 & -1 & 0 & -1 \\ 0 & 1 & \frac{1}{5} & \frac{4}{5} \\ 0 & -4 & -2 & -4 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -1 & 0 & -1 \\ 0 & 1 & \frac{1}{5} & \frac{4}{5} \\ 0 & 0 & \frac{-6}{5} & \frac{-4}{5} \end{bmatrix}$$

Row Operation 6: multiply the 3rd row by **-5/6**

$$\begin{bmatrix} 1 & -1 & 0 & -1 \\ 0 & 1 & \frac{1}{5} & \frac{4}{5} \\ 0 & 0 & \frac{-6}{5} & \frac{-4}{5} \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -1 & 0 & -1 \\ 0 & 1 & \frac{1}{5} & \frac{4}{5} \\ 0 & 0 & 1 & \frac{2}{3} \end{bmatrix}$$

9.

Row Operation 1: interchange the 1st row and the 2nd row

$$\begin{bmatrix} 0 & 1 & 3 \\ -1 & -3 & 3 \\ 1 & -3 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} -1 & -3 & 3 \\ 0 & 1 & 3 \\ 1 & -3 & 0 \end{bmatrix}$$

Row Operation 2: multiply the 1st row by **-1**

$$\begin{bmatrix} -1 & -3 & 3 \\ 0 & 1 & 3 \\ 1 & -3 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 3 & -3 \\ 0 & 1 & 3 \\ 1 & -3 & 0 \end{bmatrix}$$

Row Operation 3: add **-1** times the 1st row to the 3rd row

$$\begin{bmatrix} 1 & 3 & -3 \\ 0 & 1 & 3 \\ 1 & -3 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 3 & -3 \\ 0 & 1 & 3 \\ 0 & -6 & 3 \end{bmatrix}$$

Row Operation 4: add **6** times the 2nd row to the 3rd row

$$\begin{bmatrix} 1 & 3 & -3 \\ 0 & 1 & 3 \\ 0 & -6 & 3 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 3 & -3 \\ 0 & 1 & 3 \\ 0 & 0 & 21 \end{bmatrix}$$

Row Operation 5: multiply the 3rd row by **1/21**

$$\begin{bmatrix} 1 & 3 & -3 \\ 0 & 1 & 3 \\ 0 & 0 & 21 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 3 & -3 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix}$$

10.

Row Operation 1: multiply the 1st row by **1/3**

$$\begin{bmatrix} 3 & -2 & -3 & 3 \\ 2 & 3 & 3 & 2 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & \frac{-2}{3} & -1 & 1 \\ 2 & 3 & 3 & 2 \end{bmatrix}$$

Row Operation 2: add **-2** times the 1st row to the 2nd row

$$\begin{bmatrix} 1 & \frac{-2}{3} & -1 & 1 \\ 2 & 3 & 3 & 2 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & \frac{-2}{3} & -1 & 1 \\ 0 & \frac{13}{3} & 5 & 0 \end{bmatrix}$$

Row Operation 3: multiply the 2nd row by **3/13**

$$\begin{bmatrix} 1 & \frac{-2}{3} & -1 & 1 \\ 0 & \frac{13}{3} & 5 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & \frac{-2}{3} & -1 & 1 \\ 0 & 1 & \frac{15}{13} & 0 \end{bmatrix}$$

11.

Row Operation 1: interchange the 1st row and the 2nd row

$$\begin{bmatrix} 0 & -3 & 1 & -1 \\ -2 & 1 & 0 & 3 \end{bmatrix} \rightarrow \begin{bmatrix} -2 & 1 & 0 & 3 \\ 0 & -3 & 1 & -1 \end{bmatrix}$$

Row Operation 2: multiply the 1st row by **-1/2**

$$\begin{bmatrix} -2 & 1 & 0 & 3 \\ 0 & -3 & 1 & -1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & \frac{-1}{2} & 0 & \frac{-3}{2} \\ 0 & -3 & 1 & -1 \end{bmatrix}$$

Row Operation 3: multiply the 2nd row by **-1/3**

$$\begin{bmatrix} 1 & \frac{-1}{2} & 0 & \frac{-3}{2} \\ 0 & -3 & 1 & -1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & \frac{-1}{2} & 0 & \frac{-3}{2} \\ 0 & 1 & \frac{-1}{3} & \frac{1}{3} \end{bmatrix}$$

12.

Row Operation 1: multiply the 1st row by **-1/2**

$$\begin{bmatrix} -2 & -3 & -2 \\ 3 & -2 & -2 \\ 3 & -2 & -1 \\ -1 & -1 & -2 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & \frac{3}{2} & 1 \\ 3 & -2 & -2 \\ 3 & -2 & -1 \\ -1 & -1 & -2 \end{bmatrix}$$

Row Operation 2: add **-3** times the 1st row to the 2nd row

$$\begin{bmatrix} 1 & \frac{3}{2} & 1 \\ 3 & -2 & -2 \\ 3 & -2 & -1 \\ -1 & -1 & -2 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & \frac{3}{2} & 1 \\ 0 & \frac{-13}{2} & -5 \\ 3 & -2 & -1 \\ -1 & -1 & -2 \end{bmatrix}$$

Row Operation 3: add **-3** times the 1st row to the 3rd row

$$\begin{bmatrix} 1 & \frac{3}{2} & 1 \\ 0 & \frac{-13}{2} & -5 \\ 3 & -2 & -1 \\ -1 & -1 & -2 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & \frac{3}{2} & 1 \\ 0 & \frac{-13}{2} & -5 \\ 0 & \frac{-13}{2} & -4 \\ -1 & -1 & -2 \end{bmatrix}$$

Row Operation 4: add **1** times the 1st row to the 4th row

$$\begin{bmatrix} 1 & \frac{3}{2} & 1 \\ 0 & \frac{-13}{2} & -5 \\ 0 & \frac{-13}{2} & -4 \\ -1 & -1 & -2 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & \frac{3}{2} & 1 \\ 0 & \frac{-13}{2} & -5 \\ 0 & \frac{-13}{2} & -4 \\ 0 & \frac{1}{2} & -1 \end{bmatrix}$$

Row Operation 5: multiply the 2nd row by **-2/13**

$$\begin{bmatrix} 1 & \frac{3}{2} & 1 \\ 0 & \frac{-13}{2} & -5 \\ 0 & \frac{-13}{2} & -4 \\ 0 & \frac{1}{2} & -1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & \frac{3}{2} & 1 \\ 0 & 1 & \frac{10}{13} \\ 0 & \frac{-13}{2} & -4 \\ 0 & \frac{1}{2} & -1 \end{bmatrix}$$

Row Operation 6: add **13/2** times the 2nd row to the 3rd row

$$\begin{bmatrix} 1 & \frac{3}{2} & 1 \\ 0 & 1 & \frac{10}{13} \\ 0 & \frac{-13}{2} & -4 \\ 0 & \frac{1}{2} & -1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & \frac{3}{2} & 1 \\ 0 & 1 & \frac{10}{13} \\ 0 & 0 & 1 \\ 0 & \frac{1}{2} & -1 \end{bmatrix}$$

Row Operation 7: add **-1/2** times the 2nd row to the 4th row

$$\begin{bmatrix} 1 & \frac{3}{2} & 1 \\ 0 & 1 & \frac{10}{13} \\ 0 & 0 & 1 \\ 0 & \frac{1}{2} & -1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & \frac{3}{2} & 1 \\ 0 & 1 & \frac{10}{13} \\ 0 & 0 & 1 \\ 0 & 0 & \frac{-18}{13} \end{bmatrix}$$

Row Operation 8: add **18/13** times the 3rd row to the 4th row

$$\begin{bmatrix} 1 & \frac{3}{2} & 1 \\ 0 & 1 & \frac{10}{13} \\ 0 & 0 & 1 \\ 0 & 0 & \frac{-18}{13} \end{bmatrix} \rightarrow \begin{bmatrix} 1 & \frac{3}{2} & 1 \\ 0 & 1 & \frac{10}{13} \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

13.

Row Operation 1: add **-3** times the 1st row to the 2nd row

$$\begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & 2 \\ 1 & -3 & -1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -1 & 2 \\ 0 & 3 & -4 \\ 1 & -3 & -1 \end{bmatrix}$$

Row Operation 2: add **-1** times the 1st row to the 3rd row

$$\begin{bmatrix} 1 & -1 & 2 \\ 0 & 3 & -4 \\ 1 & -3 & -1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -1 & 2 \\ 0 & 3 & -4 \\ 0 & -2 & -3 \end{bmatrix}$$

Row Operation 3: multiply the 2nd row by **1/3**

$$\begin{bmatrix} 1 & -1 & 2 \\ 0 & 3 & -4 \\ 0 & -2 & -3 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -1 & 2 \\ 0 & 1 & \frac{-4}{3} \\ 0 & -2 & -3 \end{bmatrix}$$

Row Operation 4: add **2** times the 2nd row to the 3rd row

$$\begin{bmatrix} 1 & -1 & 2 \\ 0 & 1 & \frac{-4}{3} \\ 0 & -2 & -3 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -1 & 2 \\ 0 & 1 & \frac{-4}{3} \\ 0 & 0 & \frac{-17}{3} \end{bmatrix}$$

Row Operation 5: multiply the 3rd row by **-3/17**

$$\begin{bmatrix} 1 & -1 & 2 \\ 0 & 1 & \frac{-4}{3} \\ 0 & 0 & \frac{-17}{3} \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -1 & 2 \\ 0 & 1 & \frac{-4}{3} \\ 0 & 0 & 1 \end{bmatrix}$$

14.

Row Operation 1: $\begin{bmatrix} 3 & -3 & -2 & -1 \\ 0 & 2 & -3 & -3 \\ 3 & 3 & 2 & -3 \end{bmatrix}$ multiply the 1st row by 1/3 $\begin{bmatrix} 1 & -1 & -\frac{2}{3} & -\frac{1}{3} \\ 0 & 2 & -3 & -3 \\ 3 & 3 & 2 & -3 \end{bmatrix}$

Row Operation 2: $\begin{bmatrix} 1 & -1 & -\frac{2}{3} & -\frac{1}{3} \\ 0 & 2 & -3 & -3 \\ 3 & 3 & 2 & -3 \end{bmatrix}$ add -3 times the 1st row to the 3rd row $\begin{bmatrix} 1 & -1 & -\frac{2}{3} & -\frac{1}{3} \\ 0 & 2 & -3 & -3 \\ 0 & 6 & 4 & -2 \end{bmatrix}$

Row Operation 3: $\begin{bmatrix} 1 & -1 & -\frac{2}{3} & -\frac{1}{3} \\ 0 & 2 & -3 & -3 \\ 0 & 6 & 4 & -2 \end{bmatrix}$ multiply the 2nd row by 1/2 $\begin{bmatrix} 1 & -1 & -\frac{2}{3} & -\frac{1}{3} \\ 0 & 1 & -\frac{3}{2} & -\frac{3}{2} \\ 0 & 6 & 4 & -2 \end{bmatrix}$

Row Operation 4: $\begin{bmatrix} 1 & -1 & -\frac{2}{3} & -\frac{1}{3} \\ 0 & 1 & -\frac{3}{2} & -\frac{3}{2} \\ 0 & 6 & 4 & -2 \end{bmatrix}$ add -6 times the 2nd row to the 3rd row $\begin{bmatrix} 1 & -1 & -\frac{2}{3} & -\frac{1}{3} \\ 0 & 1 & -\frac{3}{2} & -\frac{3}{2} \\ 0 & 0 & 13 & 7 \end{bmatrix}$

Row Operation 5: $\begin{bmatrix} 1 & -1 & -\frac{2}{3} & -\frac{1}{3} \\ 0 & 1 & -\frac{3}{2} & -\frac{3}{2} \\ 0 & 0 & 13 & 7 \end{bmatrix}$ multiply the 3rd row by 1/13 $\begin{bmatrix} 1 & -1 & -\frac{2}{3} & -\frac{1}{3} \\ 0 & 1 & -\frac{3}{2} & -\frac{3}{2} \\ 0 & 0 & 1 & \frac{7}{13} \end{bmatrix}$

15.

Row Operation 1: $\begin{bmatrix} -1 & -2 & 2 \\ 2 & -2 & -2 \\ -3 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ multiply the 1st row by -1 $\begin{bmatrix} 1 & 2 & -2 \\ 2 & -2 & -2 \\ -3 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

Row Operation 2: $\begin{bmatrix} 1 & 2 & -2 \\ 2 & -2 & -2 \\ -3 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ add -2 times the 1st row to the 2nd row $\begin{bmatrix} 1 & 2 & -2 \\ 0 & -6 & 2 \\ -3 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

Row Operation 3: $\begin{bmatrix} 1 & 2 & -2 \\ 0 & -6 & 2 \\ -3 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ add 3 times the 1st row to the 3rd row $\begin{bmatrix} 1 & 2 & -2 \\ 0 & -6 & 2 \\ 0 & 6 & -5 \\ 1 & 1 & 1 \end{bmatrix}$

Row Operation 4: $\begin{bmatrix} 1 & 2 & -2 \\ 0 & -6 & 2 \\ 0 & 6 & -5 \\ 1 & 1 & 1 \end{bmatrix}$ add -1 times the 1st row to the 4th row $\begin{bmatrix} 1 & 2 & -2 \\ 0 & -6 & 2 \\ 0 & 6 & -5 \\ 0 & -1 & 3 \end{bmatrix}$

Row Operation 5: $\begin{bmatrix} 1 & 2 & -2 \\ 0 & -6 & 2 \\ 0 & 6 & -5 \\ 0 & -1 & 3 \end{bmatrix}$ multiply the 2nd row by -1/6 $\begin{bmatrix} 1 & 2 & -2 \\ 0 & 1 & -\frac{1}{3} \\ 0 & 6 & -5 \\ 0 & -1 & 3 \end{bmatrix}$

Row Operation 6: $\begin{bmatrix} 1 & 2 & -2 \\ 0 & 1 & -\frac{1}{3} \\ 0 & 6 & -5 \\ 0 & -1 & 3 \end{bmatrix}$ add -6 times the 2nd row to the 3rd row $\begin{bmatrix} 1 & 2 & -2 \\ 0 & 1 & -\frac{1}{3} \\ 0 & 0 & -3 \\ 0 & -1 & 3 \end{bmatrix}$

Row Operation 7: $\begin{bmatrix} 1 & 2 & -2 \\ 0 & 1 & -\frac{1}{3} \\ 0 & 0 & -3 \\ 0 & -1 & 3 \end{bmatrix}$ add 1 times the 2nd row to the 4th row $\begin{bmatrix} 1 & 2 & -2 \\ 0 & 1 & -\frac{1}{3} \\ 0 & 0 & -3 \\ 0 & 0 & \frac{8}{3} \end{bmatrix}$

Row Operation 8: $\begin{bmatrix} 1 & 2 & -2 \\ 0 & 1 & -\frac{1}{3} \\ 0 & 0 & -3 \\ 0 & 0 & \frac{8}{3} \end{bmatrix}$ multiply the 3rd row by -1/3 $\begin{bmatrix} 1 & 2 & -2 \\ 0 & 1 & -\frac{1}{3} \\ 0 & 0 & 1 \\ 0 & 0 & \frac{8}{3} \end{bmatrix}$

Row Operation 9: $\begin{bmatrix} 1 & 2 & -2 \\ 0 & 1 & -\frac{1}{3} \\ 0 & 0 & 1 \\ 0 & 0 & \frac{8}{3} \end{bmatrix}$ add -8/3 times the 3rd row to the 4th row $\begin{bmatrix} 1 & 2 & -2 \\ 0 & 1 & -\frac{1}{3} \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

16.

Row Operation 1: $\begin{bmatrix} 1 & -2 & -3 \\ -2 & 3 & -1 \end{bmatrix}$ add 2 times the 1st row to the 2nd row $\begin{bmatrix} 1 & -2 & -3 \\ 0 & -1 & -7 \end{bmatrix}$

Row Operation 2: $\begin{bmatrix} 1 & -2 & -3 \\ 0 & -1 & -7 \end{bmatrix}$ multiply the 2nd row by -1 $\begin{bmatrix} 1 & -2 & -3 \\ 0 & 1 & 7 \end{bmatrix}$

17.

Row Operation 1: $\begin{bmatrix} 1 & 3 & -3 & -3 \\ 0 & 0 & -2 & 2 \end{bmatrix}$ multiply the 2nd row by -1/2 $\begin{bmatrix} 1 & 3 & -3 & -3 \\ 0 & 0 & 1 & -1 \end{bmatrix}$

18.

Row Operation 1: $\begin{bmatrix} 3 & -3 & -2 \\ -2 & 1 & 1 \\ -2 & 1 & -2 \\ -1 & 1 & 2 \end{bmatrix}$ multiply the 1st row by 1/3 $\begin{bmatrix} 1 & -1 & -\frac{2}{3} \\ -2 & 1 & 1 \\ -2 & 1 & -2 \\ -1 & 1 & 2 \end{bmatrix}$

Row Operation 2: $\begin{bmatrix} 1 & -1 & -\frac{2}{3} \\ -2 & 1 & 1 \\ -2 & 1 & -2 \\ -1 & 1 & 2 \end{bmatrix}$ add 2 times the 1st row to the 2nd row $\begin{bmatrix} 1 & -1 & -\frac{2}{3} \\ 0 & -1 & -\frac{1}{3} \\ -2 & 1 & -2 \\ -1 & 1 & 2 \end{bmatrix}$

Row Operation 3: $\begin{bmatrix} 1 & -1 & -\frac{2}{3} \\ 0 & -1 & -\frac{1}{3} \\ -2 & 1 & -2 \\ -1 & 1 & 2 \end{bmatrix}$ add 2 times the 1st row to the 3rd row $\begin{bmatrix} 1 & -1 & -\frac{2}{3} \\ 0 & -1 & -\frac{1}{3} \\ 0 & -1 & -\frac{10}{3} \\ -1 & 1 & 2 \end{bmatrix}$

Row Operation 4: $\begin{bmatrix} 1 & -1 & -\frac{2}{3} \\ 0 & -1 & -\frac{1}{3} \\ 0 & -1 & -\frac{10}{3} \\ -1 & 1 & 2 \end{bmatrix}$ add 1 times the 1st row to the 4th row $\begin{bmatrix} 1 & -1 & -\frac{2}{3} \\ 0 & -1 & -\frac{1}{3} \\ 0 & -1 & -\frac{10}{3} \\ 0 & 0 & \frac{4}{3} \end{bmatrix}$

Row Operation 5: $\begin{bmatrix} 1 & -1 & -\frac{2}{3} \\ 0 & -1 & -\frac{1}{3} \\ 0 & -1 & -\frac{10}{3} \\ 0 & 0 & \frac{4}{3} \end{bmatrix}$ multiply the 2nd row by -1 $\begin{bmatrix} 1 & -1 & -\frac{2}{3} \\ 0 & 1 & \frac{1}{3} \\ 0 & -1 & -\frac{10}{3} \\ 0 & 0 & \frac{4}{3} \end{bmatrix}$

Row Operation 6: $\begin{bmatrix} 1 & -1 & -\frac{2}{3} \\ 0 & 1 & \frac{1}{3} \\ 0 & -1 & -\frac{10}{3} \\ 0 & 0 & \frac{4}{3} \end{bmatrix}$ add 1 times the 2nd row to the 3rd row $\begin{bmatrix} 1 & -1 & -\frac{2}{3} \\ 0 & 1 & \frac{1}{3} \\ 0 & 0 & -3 \\ 0 & 0 & \frac{4}{3} \end{bmatrix}$

Row Operation 7: $\begin{bmatrix} 1 & -1 & -\frac{2}{3} \\ 0 & 1 & \frac{1}{3} \\ 0 & 0 & -3 \\ 0 & 0 & \frac{4}{3} \end{bmatrix}$ multiply the 3rd row by -1/3 $\begin{bmatrix} 1 & -1 & -\frac{2}{3} \\ 0 & 1 & \frac{1}{3} \\ 0 & 0 & 1 \\ 0 & 0 & \frac{4}{3} \end{bmatrix}$

Row Operation 8: $\begin{bmatrix} 1 & -1 & -\frac{2}{3} \\ 0 & 1 & \frac{1}{3} \\ 0 & 0 & 1 \\ 0 & 0 & \frac{4}{3} \end{bmatrix}$ add -4/3 times the 3rd row to the 4th row $\begin{bmatrix} 1 & -1 & -\frac{2}{3} \\ 0 & 1 & \frac{1}{3} \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

19.

Row Operation 1: multiply the 1st row by 1/2

$$\begin{bmatrix} 2 & -3 & -2 & 3 \\ -2 & 2 & -2 & 0 \\ -1 & 3 & 3 & 2 \\ -3 & -2 & -2 & 2 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & \frac{-3}{2} & -1 & \frac{3}{2} \\ -2 & 2 & -2 & 0 \\ -1 & 3 & 3 & 2 \\ -3 & -2 & -2 & 2 \end{bmatrix}$$

Row Operation 2: add 2 times the 1st row to the 2nd row

$$\begin{bmatrix} 1 & \frac{-3}{2} & -1 & \frac{3}{2} \\ -2 & 2 & -2 & 0 \\ -1 & 3 & 3 & 2 \\ -3 & -2 & -2 & 2 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & \frac{-3}{2} & -1 & \frac{3}{2} \\ 0 & -1 & -4 & 3 \\ -1 & 3 & 3 & 2 \\ -3 & -2 & -2 & 2 \end{bmatrix}$$

Row Operation 3: add 1 times the 1st row to the 3rd row

$$\begin{bmatrix} 1 & \frac{-3}{2} & -1 & \frac{3}{2} \\ 0 & -1 & -4 & 3 \\ -1 & 3 & 3 & 2 \\ -3 & -2 & -2 & 2 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & \frac{-3}{2} & -1 & \frac{3}{2} \\ 0 & -1 & -4 & 3 \\ 0 & \frac{3}{2} & 2 & \frac{7}{2} \\ -3 & -2 & -2 & 2 \end{bmatrix}$$

Row Operation 4: add 3 times the 1st row to the 4th row

$$\begin{bmatrix} 1 & \frac{-3}{2} & -1 & \frac{3}{2} \\ 0 & -1 & -4 & 3 \\ 0 & \frac{3}{2} & 2 & \frac{7}{2} \\ -3 & -2 & -2 & 2 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & \frac{-3}{2} & -1 & \frac{3}{2} \\ 0 & -1 & -4 & 3 \\ 0 & \frac{3}{2} & 2 & \frac{7}{2} \\ 0 & \frac{-13}{2} & -5 & \frac{13}{2} \end{bmatrix}$$

Row Operation 5: multiply the 2nd row by -1

$$\begin{bmatrix} 1 & \frac{-3}{2} & -1 & \frac{3}{2} \\ 0 & -1 & -4 & 3 \\ 0 & \frac{3}{2} & 2 & \frac{7}{2} \\ 0 & \frac{-13}{2} & -5 & \frac{13}{2} \end{bmatrix} \rightarrow \begin{bmatrix} 1 & \frac{-3}{2} & -1 & \frac{3}{2} \\ 0 & 1 & 4 & -3 \\ 0 & \frac{3}{2} & 2 & \frac{7}{2} \\ 0 & \frac{-13}{2} & -5 & \frac{13}{2} \end{bmatrix}$$

Row Operation 6: add -3/2 times the 2nd row to the 3rd row

$$\begin{bmatrix} 1 & \frac{-3}{2} & -1 & \frac{3}{2} \\ 0 & 1 & 4 & -3 \\ 0 & \frac{3}{2} & 2 & \frac{7}{2} \\ 0 & \frac{-13}{2} & -5 & \frac{13}{2} \end{bmatrix} \rightarrow \begin{bmatrix} 1 & \frac{-3}{2} & -1 & \frac{3}{2} \\ 0 & 1 & 4 & -3 \\ 0 & 0 & -4 & 8 \\ 0 & \frac{-13}{2} & -5 & \frac{13}{2} \end{bmatrix}$$

Row Operation 7: add 13/2 times the 2nd row to the 4th row

$$\begin{bmatrix} 1 & \frac{-3}{2} & -1 & \frac{3}{2} \\ 0 & 1 & 4 & -3 \\ 0 & 0 & -4 & 8 \\ 0 & \frac{-13}{2} & -5 & \frac{13}{2} \end{bmatrix} \rightarrow \begin{bmatrix} 1 & \frac{-3}{2} & -1 & \frac{3}{2} \\ 0 & 1 & 4 & -3 \\ 0 & 0 & -4 & 8 \\ 0 & 0 & 21 & -13 \end{bmatrix}$$

Row Operation 8: multiply the 3rd row by -1/4

$$\begin{bmatrix} 1 & \frac{-3}{2} & -1 & \frac{3}{2} \\ 0 & 1 & 4 & -3 \\ 0 & 0 & -4 & 8 \\ 0 & 0 & 21 & -13 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & \frac{-3}{2} & -1 & \frac{3}{2} \\ 0 & 1 & 4 & -3 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 21 & -13 \end{bmatrix}$$

Row Operation 9: add -21 times the 3rd row to the 4th row

$$\begin{bmatrix} 1 & \frac{-3}{2} & -1 & \frac{3}{2} \\ 0 & 1 & 4 & -3 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 21 & -13 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & \frac{-3}{2} & -1 & \frac{3}{2} \\ 0 & 1 & 4 & -3 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 29 \end{bmatrix}$$

Row Operation 10: multiply the 4th row by 1/29

$$\begin{bmatrix} 1 & \frac{-3}{2} & -1 & \frac{3}{2} \\ 0 & 1 & 4 & -3 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 29 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & \frac{-3}{2} & -1 & \frac{3}{2} \\ 0 & 1 & 4 & -3 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

20.

Row Operation 1: multiply the 1st row by -1

$$\begin{bmatrix} -1 & 1 & -1 & -2 \\ 2 & 0 & 0 & 3 \\ -1 & 0 & -1 & -2 \\ -3 & 0 & 2 & -1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -1 & 1 & 2 \\ 2 & 0 & 0 & 3 \\ -1 & 0 & -1 & -2 \\ -3 & 0 & 2 & -1 \end{bmatrix}$$

Row Operation 2: add -2 times the 1st row to the 2nd row

$$\begin{bmatrix} 1 & -1 & 1 & 2 \\ 2 & 0 & 0 & 3 \\ -1 & 0 & -1 & -2 \\ -3 & 0 & 2 & -1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -1 & 1 & 2 \\ 0 & 2 & -2 & -1 \\ -1 & 0 & -1 & -2 \\ -3 & 0 & 2 & -1 \end{bmatrix}$$

Row Operation 3: add 1 times the 1st row to the 3rd row

$$\begin{bmatrix} 1 & -1 & 1 & 2 \\ 0 & 2 & -2 & -1 \\ -1 & 0 & -1 & -2 \\ -3 & 0 & 2 & -1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -1 & 1 & 2 \\ 0 & 2 & -2 & -1 \\ 0 & -1 & 0 & 0 \\ -3 & 0 & 2 & -1 \end{bmatrix}$$

Row Operation 4: add 3 times the 1st row to the 4th row

$$\begin{bmatrix} 1 & -1 & 1 & 2 \\ 0 & 2 & -2 & -1 \\ 0 & -1 & 0 & 0 \\ -3 & 0 & 2 & -1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -1 & 1 & 2 \\ 0 & 2 & -2 & -1 \\ 0 & -1 & 0 & 0 \\ 0 & -3 & 5 & 5 \end{bmatrix}$$

Row Operation 5: multiply the 2nd row by 1/2

$$\begin{bmatrix} 1 & -1 & 1 & 2 \\ 0 & 2 & -2 & -1 \\ 0 & -1 & 0 & 0 \\ 0 & -3 & 5 & 5 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -1 & 1 & 2 \\ 0 & 1 & -1 & \frac{-1}{2} \\ 0 & -1 & 0 & 0 \\ 0 & -3 & 5 & 5 \end{bmatrix}$$

Row Operation 6: add 1 times the 2nd row to the 3rd row

$$\begin{bmatrix} 1 & -1 & 1 & 2 \\ 0 & 1 & -1 & \frac{-1}{2} \\ 0 & -1 & 0 & 0 \\ 0 & -3 & 5 & 5 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -1 & 1 & 2 \\ 0 & 1 & -1 & \frac{-1}{2} \\ 0 & 0 & -1 & \frac{-1}{2} \\ 0 & -3 & 5 & 5 \end{bmatrix}$$

Row Operation 7: add 3 times the 2nd row to the 4th row

$$\begin{bmatrix} 1 & -1 & 1 & 2 \\ 0 & 1 & -1 & \frac{-1}{2} \\ 0 & 0 & -1 & \frac{-1}{2} \\ 0 & -3 & 5 & 5 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -1 & 1 & 2 \\ 0 & 1 & -1 & \frac{-1}{2} \\ 0 & 0 & -1 & \frac{-1}{2} \\ 0 & 0 & 2 & \frac{7}{2} \end{bmatrix}$$

Row Operation 8: multiply the 3rd row by -1

$$\begin{bmatrix} 1 & -1 & 1 & 2 \\ 0 & 1 & -1 & \frac{-1}{2} \\ 0 & 0 & -1 & \frac{-1}{2} \\ 0 & 0 & 2 & \frac{7}{2} \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -1 & 1 & 2 \\ 0 & 1 & -1 & \frac{-1}{2} \\ 0 & 0 & 1 & \frac{1}{2} \\ 0 & 0 & 2 & \frac{7}{2} \end{bmatrix}$$

Row Operation 9: add -2 times the 3rd row to the 4th row

$$\begin{bmatrix} 1 & -1 & 1 & 2 \\ 0 & 1 & -1 & \frac{-1}{2} \\ 0 & 0 & 1 & \frac{1}{2} \\ 0 & 0 & 2 & \frac{7}{2} \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -1 & 1 & 2 \\ 0 & 1 & -1 & \frac{-1}{2} \\ 0 & 0 & 1 & \frac{1}{2} \\ 0 & 0 & 0 & \frac{5}{2} \end{bmatrix}$$

Row Operation 10: multiply the 4th row by 2/5

$$\begin{bmatrix} 1 & -1 & 1 & 2 \\ 0 & 1 & -1 & \frac{-1}{2} \\ 0 & 0 & 1 & \frac{1}{2} \\ 0 & 0 & 0 & \frac{5}{2} \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -1 & 1 & 2 \\ 0 & 1 & -1 & \frac{-1}{2} \\ 0 & 0 & 1 & \frac{1}{2} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

21.

Row Operation 1: $\begin{bmatrix} -3 & -1 & 3 & -3 \\ 0 & -2 & -1 & 1 \\ 3 & 2 & -3 & 0 \end{bmatrix}$ multiply the 1st row by **-1/3** $\begin{bmatrix} 1 & \frac{1}{3} & -1 & 1 \\ 0 & -2 & -1 & 1 \\ 3 & 2 & -3 & 0 \end{bmatrix}$

Row Operation 2: $\begin{bmatrix} 1 & \frac{1}{3} & -1 & 1 \\ 0 & -2 & -1 & 1 \\ 3 & 2 & -3 & 0 \end{bmatrix}$ add **-3** times the 1st row to the 3rd row $\begin{bmatrix} 1 & \frac{1}{3} & -1 & 1 \\ 0 & -2 & -1 & 1 \\ 0 & 1 & 0 & -3 \end{bmatrix}$

Row Operation 3: $\begin{bmatrix} 1 & \frac{1}{3} & -1 & 1 \\ 0 & -2 & -1 & 1 \\ 0 & 1 & 0 & -3 \end{bmatrix}$ multiply the 2nd row by **-1/2** $\begin{bmatrix} 1 & \frac{1}{3} & -1 & 1 \\ 0 & 1 & \frac{1}{2} & -\frac{1}{2} \\ 0 & 1 & 0 & -3 \end{bmatrix}$

Row Operation 4: $\begin{bmatrix} 1 & \frac{1}{3} & -1 & 1 \\ 0 & 1 & \frac{1}{2} & -\frac{1}{2} \\ 0 & 1 & 0 & -3 \end{bmatrix}$ add **-1** times the 2nd row to the 3rd row $\begin{bmatrix} 1 & \frac{1}{3} & -1 & 1 \\ 0 & 1 & \frac{1}{2} & -\frac{1}{2} \\ 0 & 0 & -\frac{1}{2} & -\frac{5}{2} \end{bmatrix}$

Row Operation 5: $\begin{bmatrix} 1 & \frac{1}{3} & -1 & 1 \\ 0 & 1 & \frac{1}{2} & -\frac{1}{2} \\ 0 & 0 & -\frac{1}{2} & -\frac{5}{2} \end{bmatrix}$ multiply the 3rd row by **-2** $\begin{bmatrix} 1 & \frac{1}{3} & -1 & 1 \\ 0 & 1 & \frac{1}{2} & -\frac{1}{2} \\ 0 & 0 & 1 & 5 \end{bmatrix}$

22.

Row Operation 1: $\begin{bmatrix} 2 & 1 \\ -1 & 1 \\ 0 & -2 \\ 1 & 3 \end{bmatrix}$ multiply the 1st row by **1/2** $\begin{bmatrix} 1 & \frac{1}{2} \\ -1 & 1 \\ 0 & -2 \\ 1 & 3 \end{bmatrix}$

Row Operation 2: $\begin{bmatrix} 1 & \frac{1}{2} \\ -1 & 1 \\ 0 & -2 \\ 1 & 3 \end{bmatrix}$ add **1** times the 1st row to the 2nd row $\begin{bmatrix} 1 & \frac{1}{2} \\ 0 & \frac{3}{2} \\ 0 & -2 \\ 1 & 3 \end{bmatrix}$

Row Operation 3: $\begin{bmatrix} 1 & \frac{1}{2} \\ 0 & \frac{3}{2} \\ 0 & -2 \\ 1 & 3 \end{bmatrix}$ add **-1** times the 1st row to the 4th row $\begin{bmatrix} 1 & \frac{1}{2} \\ 0 & \frac{3}{2} \\ 0 & -2 \\ 0 & \frac{5}{2} \end{bmatrix}$

Row Operation 4: $\begin{bmatrix} 1 & \frac{1}{2} \\ 0 & \frac{3}{2} \\ 0 & -2 \\ 0 & \frac{5}{2} \end{bmatrix}$ multiply the 2nd row by **2/3** $\begin{bmatrix} 1 & \frac{1}{2} \\ 0 & 1 \\ 0 & -2 \\ 0 & \frac{5}{2} \end{bmatrix}$

Row Operation 5: $\begin{bmatrix} 1 & \frac{1}{2} \\ 0 & 1 \\ 0 & -2 \\ 0 & \frac{5}{2} \end{bmatrix}$ add **2** times the 2nd row to the 3rd row $\begin{bmatrix} 1 & \frac{1}{2} \\ 0 & 1 \\ 0 & 0 \\ 0 & \frac{5}{2} \end{bmatrix}$

Row Operation 6: $\begin{bmatrix} 1 & \frac{1}{2} \\ 0 & 1 \\ 0 & 0 \\ 0 & \frac{5}{2} \end{bmatrix}$ add **-5/2** times the 2nd row to the 4th row $\begin{bmatrix} 1 & \frac{1}{2} \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$

23.

Row Operation 1: $\begin{bmatrix} 0 & -3 & 2 & -2 \\ 0 & 2 & 2 & -2 \end{bmatrix}$ multiply the 1st row by **-1/3** $\begin{bmatrix} 0 & 1 & -\frac{2}{3} & \frac{2}{3} \\ 0 & 2 & 2 & -2 \end{bmatrix}$

Row Operation 2: $\begin{bmatrix} 0 & 1 & -\frac{2}{3} & \frac{2}{3} \\ 0 & 2 & 2 & -2 \end{bmatrix}$ add **-2** times the 1st row to the 2nd row $\begin{bmatrix} 0 & 1 & -\frac{2}{3} & \frac{2}{3} \\ 0 & 0 & \frac{10}{3} & -\frac{10}{3} \end{bmatrix}$

Row Operation 3: $\begin{bmatrix} 0 & 1 & -\frac{2}{3} & \frac{2}{3} \\ 0 & 0 & \frac{10}{3} & -\frac{10}{3} \end{bmatrix}$ multiply the 2nd row by **3/10** $\begin{bmatrix} 0 & 1 & -\frac{2}{3} & \frac{2}{3} \\ 0 & 0 & 1 & -1 \end{bmatrix}$

24.

Row Operation 1: $\begin{bmatrix} -2 & -3 \\ -1 & -1 \\ -3 & -3 \end{bmatrix}$ multiply the 1st row by **-1/2** $\begin{bmatrix} 1 & \frac{3}{2} \\ -1 & -1 \\ -3 & -3 \end{bmatrix}$

Row Operation 2: $\begin{bmatrix} 1 & \frac{3}{2} \\ -1 & -1 \\ -3 & -3 \end{bmatrix}$ add **1** times the 1st row to the 2nd row $\begin{bmatrix} 1 & \frac{3}{2} \\ 0 & \frac{1}{2} \\ -3 & -3 \end{bmatrix}$

Row Operation 3: $\begin{bmatrix} 1 & \frac{3}{2} \\ 0 & \frac{1}{2} \\ -3 & -3 \end{bmatrix}$ add **3** times the 1st row to the 3rd row $\begin{bmatrix} 1 & \frac{3}{2} \\ 0 & \frac{1}{2} \\ 0 & \frac{3}{2} \end{bmatrix}$

Row Operation 4: $\begin{bmatrix} 1 & \frac{3}{2} \\ 0 & \frac{1}{2} \\ 0 & \frac{3}{2} \end{bmatrix}$ multiply the 2nd row by **2** $\begin{bmatrix} 1 & \frac{3}{2} \\ 0 & 1 \\ 0 & \frac{3}{2} \end{bmatrix}$

Row Operation 5: $\begin{bmatrix} 1 & \frac{3}{2} \\ 0 & 1 \\ 0 & \frac{3}{2} \end{bmatrix}$ add **-3/2** times the 2nd row to the 3rd row $\begin{bmatrix} 1 & \frac{3}{2} \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$

25.

Row Operation 1:

$$\begin{bmatrix} -3 & 3 & -2 \\ 2 & 2 & 3 \\ 1 & 1 & 0 \end{bmatrix} \xrightarrow{\text{multiply the 1st row by } -1/3} \begin{bmatrix} 1 & -1 & \frac{2}{3} \\ 2 & 2 & 3 \\ 1 & 1 & 0 \end{bmatrix}$$

Row Operation 2:

$$\begin{bmatrix} 1 & -1 & \frac{2}{3} \\ 2 & 2 & 3 \\ 1 & 1 & 0 \end{bmatrix} \xrightarrow{\text{add } -2 \text{ times the 1st row to the 2nd row}} \begin{bmatrix} 1 & -1 & \frac{2}{3} \\ 0 & 4 & \frac{5}{3} \\ 1 & 1 & 0 \end{bmatrix}$$

Row Operation 3:

$$\begin{bmatrix} 1 & -1 & \frac{2}{3} \\ 0 & 4 & \frac{5}{3} \\ 1 & 1 & 0 \end{bmatrix} \xrightarrow{\text{add } -1 \text{ times the 1st row to the 3rd row}} \begin{bmatrix} 1 & -1 & \frac{2}{3} \\ 0 & 4 & \frac{5}{3} \\ 0 & 2 & \frac{-2}{3} \end{bmatrix}$$

Row Operation 4:

$$\begin{bmatrix} 1 & -1 & \frac{2}{3} \\ 0 & 4 & \frac{5}{3} \\ 0 & 2 & \frac{-2}{3} \end{bmatrix} \xrightarrow{\text{multiply the 2nd row by } 1/4} \begin{bmatrix} 1 & -1 & \frac{2}{3} \\ 0 & 1 & \frac{5}{12} \\ 0 & 2 & \frac{-2}{3} \end{bmatrix}$$

Row Operation 5:

$$\begin{bmatrix} 1 & -1 & \frac{2}{3} \\ 0 & 1 & \frac{5}{12} \\ 0 & 2 & \frac{-2}{3} \end{bmatrix} \xrightarrow{\text{add } -2 \text{ times the 2nd row to the 3rd row}} \begin{bmatrix} 1 & -1 & \frac{2}{3} \\ 0 & 1 & \frac{5}{12} \\ 0 & 0 & \frac{-3}{2} \end{bmatrix}$$

Row Operation 6:

$$\begin{bmatrix} 1 & -1 & \frac{2}{3} \\ 0 & 1 & \frac{5}{12} \\ 0 & 0 & \frac{-3}{2} \end{bmatrix} \xrightarrow{\text{multiply the 3rd row by } -2/3} \begin{bmatrix} 1 & -1 & \frac{2}{3} \\ 0 & 1 & \frac{5}{12} \\ 0 & 0 & 1 \end{bmatrix}$$