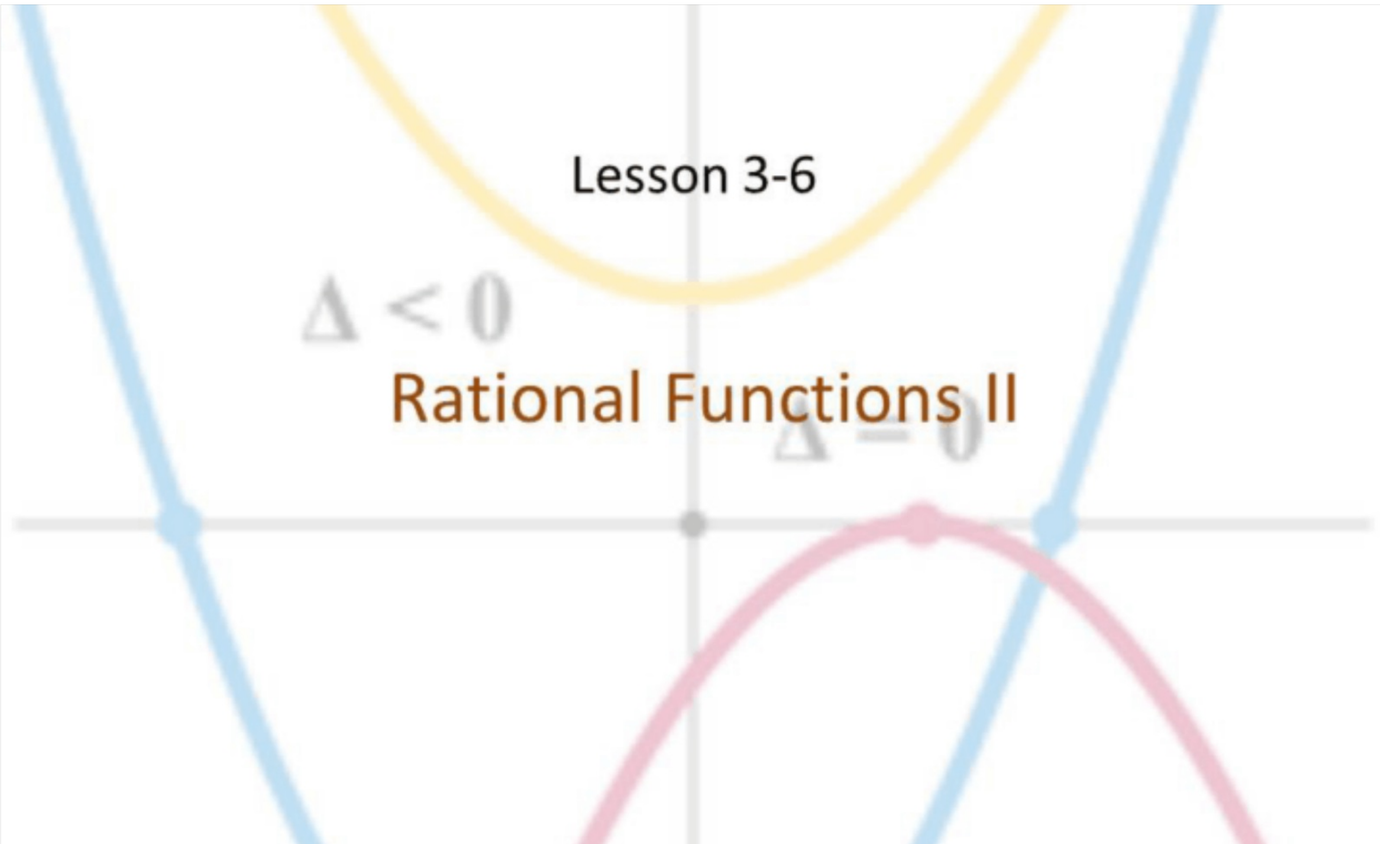
Lesson 3-6

Rational Functions II



Objective

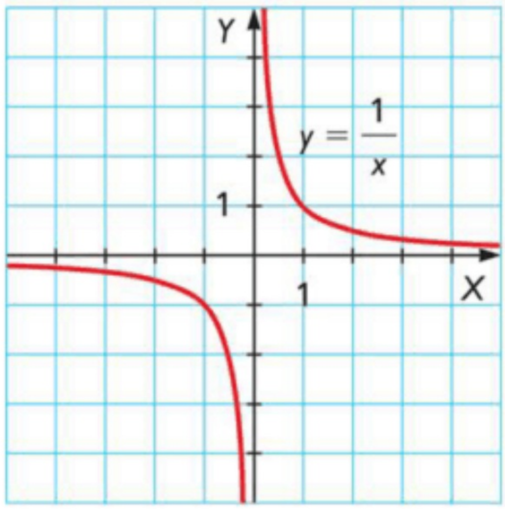
Students will…

- Be able to identify and solve for both vertical and horizontal asymptotes of rational functions.
- Be able to find the slant or oblique asymptote of a rational function, given that it exists.

Asymptotes

One of the characteristics of rational function graphs is the presence of asymptotes. **Asymptotes** are lines that the graph of the function gets closer and closer to as it travels on. You may think of them as invisible boundary lines that the graph continually approaches.

ex. $f(x) = \frac{1}{x}$



We can see that both x and the y-axis are asymptotes of this graph.

Vertical Asymptotes

From the previous graph, we saw that there were two different types of asymptotes at play. There was a **vertical** asymptotes (the y-axis), as well as a **horizontal** asymptote (the x-axis). So for every rational function graph, we must consider both.

Recall that the vertical lines represent the horizontal or the x-coordinates. **Thus, to find vertical asymptotes, we must consider the possible x-coordinates that would make the rational functions undefined, i.e. what x-value makes the denominator 0?**

ex. $f(x) = \frac{1}{x}$ For this function, it's obvious that the only place the function is undefined would be when $x = 0$, which is the y-axis. Therefore, it becomes the **vertical asymptotes**.

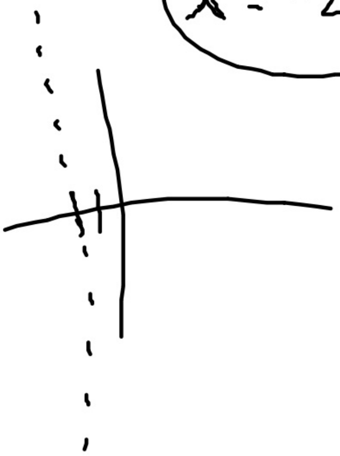
Examples

Find the vertical asymptotes of the following functions.

1. $f(x) = \frac{x-6}{x+2}$ 2. $g(x) = \frac{8}{2x-9}$ 3. $h(x) = \frac{x-9}{5}$

Vert. asymp: $x+2=0$

$x=-2$



$2x-9=0$

$x=9/2$

Vert. asymp: None

Horizontal Asymptotes

$$\frac{x^2+1}{x+11}$$

Horizontal asymptotes are horizontal lines, which represents a certain y-value (y=...). The method for finding horizontal asymptotes is as follows:

Let n be the leading exponent of the numerator and m be the leading exponent of the denominator.

(a). If $n < m$, i.e. higher degree in the denominator, the horizontal asymptotes is $y = 0$.

$y = 3/2$

(b). If $n = m$, then the horizontal asymptote is $y = \frac{coefficient\ of\ leading\ term}{coefficient\ of\ leading\ term}$ $\frac{\boxed{3}x-1}{\boxed{2}x+13}$

(c). If $n > m$, i.e. higher degree in the numerator, then no horizontal asymptote exists

Examples

Find the horizontal asymptotes of the following functions.

1. $f(x) = \frac{x^2-6}{x^3+2}$ 2. $g(x) = \frac{8x}{2x-9}$ 3. $h(x) = \frac{9x^4}{5}$

horiz. asymp: $y=0$

horiz asymp: $y = \frac{8}{2} = 4$

horiz asympt: None

Slant or Oblique Asymptotes

For situations where no horizontal asymptotes exist, i.e. higher degree in the numerator, there may still exist a **slant or oblique** asymptote. Finding such asymptote is a rather easy process, as it is simply done by dividing (long division is needed here).

$$\text{Ex. } f(x) = \frac{x^2-4x-5}{x-3}$$

Here we can easily see that no horizontal asymptote exists. We just need to divide to see if there is a slant or oblique asymptote.

$$\begin{array}{r l}
 & x-1 \\
x-3 \,\big) & x^2-4x-5 \\
\ominus & x^2-3x \quad \downarrow \\
\hline
 & \quad -x-5 \\
 & \quad -x+3 \\
\hline
 & \quad \vdots
\end{array}$$

$$\boxed{y = x-1}\,.$$

Examples

Find the asymptotes of the following functions. If no horizontal asymptote exists, find slant or oblique asymptotes.

1. $f(x) = \frac{5x+21}{x^2+10x+25}$

Vert: $x^2+10x+25=0$

$(x+5)(x+5)=0$

$x=-5$

horiz: $y=0$

2. $f(x) = \frac{x^3+3x^2}{x^2-4}$

Vert.: $x^2-4=0$

$\sqrt{x^2}=\sqrt{4}$

$\boxed{x=\pm 2}$

Horiz: None

Slant: $\boxed{y=x+3}$

$$\begin{array}{r} x+3 \\ x^2-4\,\overline{)\,x^3+3x^2+0x} \\ -\;(x^3-4x) \\ \hline 3x^2+4x \\ 3x^2-12 \\ \hline \end{array}$$

Examples

3. $f(x) = \frac{x^2-3x-4}{2x^2+4x}$

Vert: $2x^2+4x=0$

$2x(x+2)=0$

$\boxed{x=-2,0}$

Horiz: $\boxed{y=1/2}$

4. $f(x) = \frac{2x^2+7x-4}{x^2+x-2}$

Vert: $\boxed{x=-2,1}$

horiz: $\boxed{y=\frac{2}{1}=2}$

Homework 10/30

TB pg. 313 #11-23 (odd)