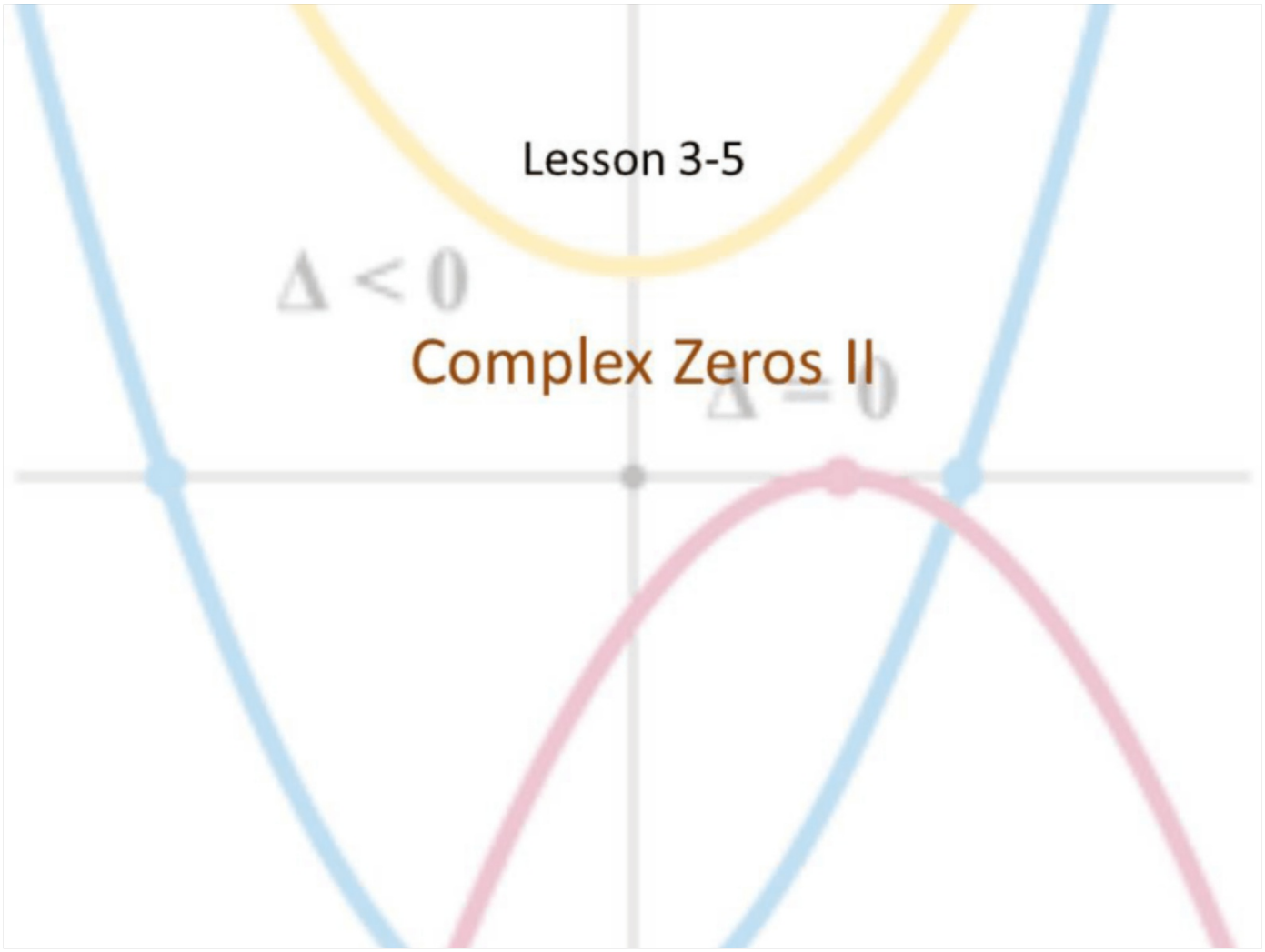
Lesson 3-5
$\Delta < 0$
Complex Zeros II
$\Delta = 0$

Objective

Students will...

- Be able to find polynomials with specified zeros.
- Be able to understand what Conjugate Zeros Theorem says and use it to find polynomials.

Finding Polynomials with Specified Zeros

We have been learning how to factor a polynomial in order to find its zeros. The backward process can also be done. Consider a polynomial *P* with zeros 0 and -3. Based on what we know about factored polynomials. These zeros can be derived from $P(x) = x(x + 3)$

or. $2x(x+3)$
or. $x^2(x+3)$
or. $x(x+3)^3$

So, when we multiply it out, $P(x) = x^2 + 3x$

Hence, if the degree of the polynomial is known, along with its zeros, we can derive the original function.

Example

Find a polynomial $Q(x)$ of degree 4, with zeros -2 and 0, where -2 is a zero of multiplicity 3. **Note:** $(A+B)^3 = (A^3 + 3A^2B + 3AB^2 + B^3)$

$Q(x) = x(\overset{A}{x} + \overset{B}{2})^3 = x(x^3 + 6x^2 + 12x + 8).$

$Q(x) = x^4 + 6x^3 + 12x^2 + 8x$

Example

Find a polynomial $P(x)$ of degree 4, with zeros $i, -i, 2$, and -2.

$$P(x) = (x-i)(x+i)(x-2)(x+2)$$

$$= (x^2 - i^2)(x^2 - 4) = (x^2+1)(x^2-4)$$

$$= x^4 - 4x^2 + x^2 - 4 \Rightarrow \boxed{P(x) = x^4 - 3x^2 - 4}.$$

Conjugate Pairs

There is an interesting thing to observe regarding conjugates of complex numbers.

ex. $3 = 3 + 0i$

$3 - 0i$

Conjugate Zeros Theorem- If the polynomial P has real coefficients, and if the complex number z is a zero of P, then its complex conjugate is also a zero of P.

In other words, if a certain $a + bi$ is a zero (x-intercept) of a polynomial, then its conjugate, $a - bi$ is also a zero.

Example

Find a polynomial $P(x)$ of degree 3 that has integer coefficients and zeros $\frac{1}{2}$ and $3-i$. , $3+i$

$$P(x) = (2x-1)(x-(3-i))(x-(3+i)).$$

$$P(x) = (2x-1)(x-3+i)(x-3-i).$$

$$= (2x-1)(x^2-3x-ix-3x+9+3i+ix-3i-i^2)$$

$$= (2x-1)(x^2-6x+10) = 2x^3-12x^2+20x-x^2+6x-10$$

$$\boxed{P(x) = 2x^3-13x^2+26x-10}$$

Homework 10/28

TB pg. 298 #31-37 (odd), 41, 46