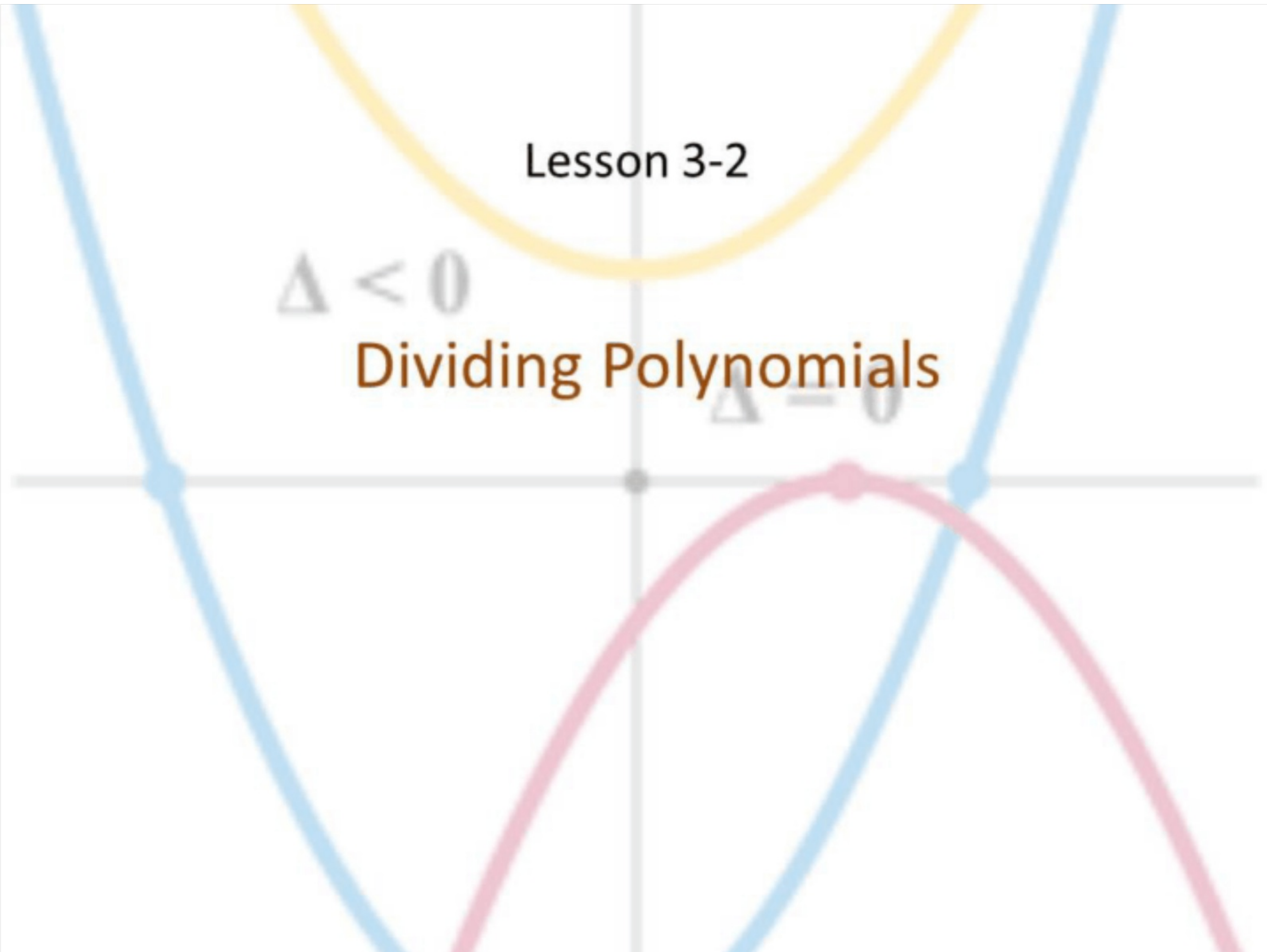
Lesson 3-2

Dividing Polynomials



Objective

Students will…

- Be able to use long division and synthetic division to divide a polynomial.
- Be able to identify the dividend, divisor, quotient, and the remainder after dividing a polynomial using long division.
- Be able to know and use the Remainder and Factor Theorem.

Studying Polynomials Algebraically

Polynomials can also be studied extensively using algebra. From our experience with graphing, we realized that factoring is a powerful tool when it comes to studying polynomials. Recall that factoring really is dividing. Hence, to factor, we must learn how to divide polynomials.

Ex. x^3+3x^2-4x

$x(x^2+3x-4)$

Dividing polynomials is much like dividing numbers.

$$\begin{array}{r} 5 \text{ R}3 \\ 7\overline{)38} \\ -35 \\ \hline 3 \end{array}$$

$\boxed{5\ 3/7}$

Ex. $\frac{38}{7} = 5 + \frac{3}{7}$

When we divide 38 by 7, we end up with a quotient of 5 and the remainder of $\frac{3}{7}$

Long Division

41

3 | 123

- 12

03

Much like how we first learned how to divide numbers, we can use long division to divide polynomials.

Example: ~~Divide~~ Divide $6x^2 - 26x + 12$ by $x - 4$.

$6x - 2$

$x - 4 \,\overline{)\, 6x^2 - 26x + 12}$

$-\;\; 6x^2 - 24x$

$-2x + 12$

$-\;\; -2x + 8$

4 ← Remainder

Long Division

So, we are done when the long division ends with a polynomial that is of <u>lesser degree</u> than what we divided by. In our case, we divided by $x - 4$ (degree 1) and ended up with 4 (degree 0). So, we can interpret our result in two ways:

Quotient

$$\frac{6x^2 - 26x + 12}{x - 4} = 6x - 2 + \frac{4}{x - 4}$$

Ex.

$$\frac{10}{3} = 3 + \frac{1}{3}$$

Or

$$6x^2 - 26x + 12 = (x - 4)(6x - 2) + 4$$

Ex.

$$10 = 3 \times 3 + 1$$

$3a = 2$ $\quad a = 2/3$ $\qquad 3a = 1$ $\quad a = 1/3$ $\qquad 2a = -7$ $\quad a = -7/2$

$\frac{40}{2} - \frac{21}{2} = 19/2$

Example

$2x^5 - 7x^4 - 13 = \frac{1}{2}\left(x^3 - 2x^2 - 5x - 7/2\right)\left(2x^2 - 3x \ldots\right) + 19/2\,x \ldots$

For the following, find the quotient and remainder using long division.

1. $\frac{x^2-6x-8}{x-4}$

$x - 2$

$x-4 \,)\, x^2 - 6x - 8$

$- \quad x^2 - 4x$

$-2x - 8$

$- \quad -2x + 8$

-16

2. $\frac{x^3+3x^2+4x+3}{3x+6}$

$\frac{1}{3}x^2 + \frac{1}{3}x + 2/3$

$3x+6 \,)\, x^3 + 3x^2 + 4x + 3$

$- \quad x^3 + 2x^2$

$x^2 + 4x$

$- \quad x^2 + 2x$

$2x + 3$

$- \quad 2x + 4$

$\boxed{-1}$

3. $\frac{2x^5-7x^4-13}{4x^2-6x+8} = \frac{1}{2} \cdot \frac{2x^5 - 7 \ldots}{2x^2 - 3 \ldots}$

$x^3 - 2x^2 - 5x - 7/2$

$2x^2 - 3x + 4 \,)\, 2x^5 - 7x^4 - 13 + 0x^3$

$- \quad 2x^5 - 3x^4 + 4x^3$

$-4x^4 - 4x^3 + 0x^2$

$- \quad -4x^4 + 6x^3 - 8x^2$

$-10x^3 + 8x^2 + 0x$

$- \quad -10x^3 + 15x^2 - 20x$

$-7x^2 + 20x - 13$

$- \quad -7x^2 + \frac{21}{2}x - 14$

$\boxed{19/2\,x + 1}$

Synthetic Division

Although long division will always get the job done with dividing polynomials, synthetic division is a quicker method. The only drawback to synthetic division is that it can only be used when the divisor is of the form $\boxed{x - c.}$ Here are few things to keep in mind when using synthetic division:

- We only need to use the coefficient of each term.
- Need to make sure to include “0” in places where a degree term is missing. For example, for polynomial $x^3 + x - 7$, the coefficient we use would be 1, 0, 1, -7 (0 for the degree 2 term).
- For our divisor $x - c$ the constant we use as our divisor is $-c$. For example, for divisor $x - 8$, the constant we use would be $-(-8) = 8$.

$(x-8)(x-3) = 0$

$x = 8, 3$

Example

$+0x$

Divide $2x^3 - 7x^2 + 5$ by $x - 3$ using synthetic division.

$$
\begin{array}{c|cccc}
3 & 2 & -7 & 0 & 5 \\
+ & \downarrow & 6 & -3 & -9 \\
\hline
 & 2 & -1 & -3 & \boxed{-4}
\end{array}
= P(3)
$$

$$2x^3 - 7x^2 + 5 = (x-3)(2x^2 - x - 3) - 4$$

Example

Use synthetic division to divide $\mathrm{P(x)} = 5x^3 - 2x^2 + x - 10$ by $x - 3$

Remainder Theorem

Synthetic division is useful because it can sometimes cut time on evaluating polynomials.

<u>The Remainder Theorem</u>- If polynomial $P(x)$ is divided by $x - c$, then the remainder is the value of $P(c)$.

So, applying to our previous example, we know that 110 (the remainder) is the value of $P(3)$.

Example

Let $P(x) = 3x^5 + 5x^4 - 4x^3 + 7x + 3$. Divide by $x + 2$ using synthetic division. Then, use the remainder theorem to evaluate $P(-2)$.

$$
\begin{array}{r|rrrrrr}
-2 & 3 & 5 & -4 & 0 & 7 & 3 \\
+ & \downarrow & -6 & 2 & 4 & -8 & 2 \\
\hline
 & 3 & -1 & -2 & 4 & -1 & \boxed{5}
\end{array}
$$

$$\boxed{P(-2) = 5}$$

Factor Theorem

The last theorem to observe in this section is Factor Theorem, which says that zeros of polynomials correspond to factors.

Polynomial

complete.

The Factor Theorem- c is a zero of P if and only if $x - c$ is a factor of $P(x)$.

So, if a certain c is a zero of any polynomial, performing either long division or synthetic division by the divisor $x - c$ should yield **no** remainder (or remainder 0).

$$x^2 + 2x + 1 = 0$$

$$(x+1)(x+1) = 0$$

$$x = -1$$

Example

$x-1$

Use the Factor Theorem to show that $x - c$ is a factor of $P(x) = x^3 - 3x^2 + 3x - 1$, $c = 1$ and factor completely to find all the zeros.

$$
\begin{array}{c|cccc}
1 & 1 & -3 & 3 & -1 \\
+ & \downarrow & 1 & -2 & 1 \\
\hline
 & 1 & -2 & 1 & \boxed{0}
\end{array}
$$

$$P(x) = (x-1)(x^2-2x+1) = (x-1)(x-1)(x-1) = \boxed{(x-1)^3}$$

Homework 10/16

TB pg. 270-271 #13-53 (e.o.o)